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A CULTURAL APPROACH TO ELEVENTH GRADE TRIGONOMETRY

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "A Cultural Approach to Eleventh Grade Trigonometry" submitted by Gordon O. Ellis in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of the study was to develop a unit of mathematics from a cultural perspective and to investigate the resultant cognitive and affective outcomes of instruction on students and their learning.

The major characteristics of the cultural approach to mathematics instruction were identified by reviewing the related literature. Based on these characteristics, lesson objectives and instructional support materials for a unit on trigonometry were developed for classroom use.

Four classes, two from each of two schools, and two teachers participated in the study. In each school one class was taught the unit from the cultural approach while the other was taught the unit in the regular manner. Data were collected by achievement tests, questionnaires and debriefing interviews.

The results of the study indicated no significant difference in the mean of the unit achievement scores between students in the cultural classes and students in the regular classes. Students in the cultural classes found the unit to be more difficult and perceived trigonometry to be more useful than did students in the regular classes. In response to the debriefing interviews, students in the cultural classes recalled information of a more complex nature concerning their learning, than did students in the regular classes. Students in the cultural

classes expressed a more positive attitude toward the unit than did students in the regular classes.

This study indicated that it is feasible for classroom teachers to incorporate material of a cultural nature into mathematics classes and to obtain positive results.

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THE PROBLEM

I. BACKGROUND AND SIGNIFICANCE OF THE STUDY

Major changes in the content, scope and sequence of secondary school mathematics curricula took place during the late fifties and early sixties. The new courses were well researched and designed, reflecting the best thinking of professional mathematicians and mathematics educators of North America. The product of this effort was a new curriculum in mathematics, reflecting more adequately than before the unity among previously disjoint topics by emphasizing, among other things, the centrality of sets, functions, transformations and the importance of deductive thought and structure in the teaching and learning of mathematics.

Yet in spite of the unprecedented prominence that mathematics has held in school curricula for some twenty years, today strident criticism from within and without the educational community has forced many mathematics educators to re-examine the wisdom and effectiveness of their varied innovations. Cries for change are emerging from many sectors of society. Parents and employers advocate a "return to the basics," however ill-defined the "basics" may be. Teachers insist that the existing courses are

excessively formal, deductively structured and theoretical, reflecting the theoretical mathematical training and bias of the curriculum designers. They sense too that the courses have tended to isolate mathematics from its meaningful applications to the real world. Coleman, Edwards and Beltzner (1975) point to the student and his concerns, for they observe: "Students today find their high school encounter with mathematics a deeply frustrating experience. They can neither enjoy it as a thing in itself, nor can they relate to it anything which they do enjoy" (p. 53).

Care must be taken to appraise these criticisms realistically, for as Hill et al. (1975) point out, "all of American education has been deeply conditioned by recent cultural changes, school-work attitudes, habits and motivations have dramatically altered and, as a consequence, have definitely reduced many formerly expected educational outcomes" (p. 147). Coleman, Edwards and Beltzner (1975) have made similar observations concerning the Canadian scene for they state "the new curriculum has been accompanied by very great changes in attitudes among students, changes in their study-habits and an increasingly affluent society . . ." (p. 97). These statements may imply that mathematics educators should turn their energies more to the attainment of the affective goals of instruction than they have in the past. Nevertheless, the overt displeasure with mathematics education as it exists today

is not imaginary. The problems are real and are presently being studied extensively at all levels of the educational spectrum.

One of the most severe and influential critics of the modern program has been Morris Kline (1973) of New York University. He has argued for twenty years that the new programs have isolated mathematics from its creators, its historical development, other disciplines of human knowledge and its significant role in and connection with the physical and social world of the student. He claims that as a result, mathematics has been dehumanized and misrepresented, leaving students with a distorted view of what mathematics really is.

More recently, other educators such as May (1971), Higginson (1973), Howson (1973), Edwards (1973) and Coleman, Edwards and Beltzner (1975) have adopted and actively advocated many of Kline's proposals. These include the practice of embedding a mathematical topic wherever possible in its historical context; putting the mathematicians back into mathematics; allowing students to consider and solve problems of historical interest that are within their grasp and related to the topic; pointing out the evolution of the concept, topic or symbol through time; providing students with practical problems that are of interest to them and relevant to their world, and providing information and experience which indicate

clearly the power, utility and importance of mathematics and mathematical methods in the efficient functioning of modern society. An approach to mathematics instruction which incorporates these characteristics in the regular teaching strategy is called the "cultural approach."

Those who advocate the cultural approach to mathematics instruction suggest several benefits which should accrue as a result of such an instructional strategy. Most of these relate to the affective rather than the cognitive goals of mathematics instruction. Advocates suggest that if students learn mathematics within a meaningful context and come to recognize the central importance of the discipline in their lives and their culture, they will come to see mathematics as more meaningful, interesting and relevant to their existence. Some suggest, rather cautiously, that students may, as a consequence of improved attitudes toward the subject, be motivated to further study, greater understanding or higher achievement.

There exists a need to investigate the feasibility of designing and implementing an instructional unit based on the ideals outlined above and to examine the educational outcomes of such an undertaking.

II. STATEMENT OF THE PROBLEM

The purpose of this study is to design and produce classroom materials for a unit on trigonometry from a cultural perspective, to teach the unit in a school setting and to seek answers to the following questions:

1. Are there differences in achievement between students under the cultural treatment and students under the "regular" treatment? (See page 6 for the definition of "regular" treatment.)

2. Are there differences in certain affective outcomes between students under the cultural treatment and students under the regular treatment?

3. Are there differences in the type and complexity of information about the trigonometry unit recalled by students under the cultural treatment and those under the regular treatment?

III. DEFINITION OF TERMS

Culture: The sum total of the attainments and learned behavior patterns of any specific period, race, or people, regarded as expressing a traditional way of life subject to gradual but continuous modification by succeeding generations.

Cultural Approach to Mathematics Instruction: An approach which emphasizes the natural and historic inter-relationship of mathematics with other areas of human

knowledge, activities and interests. It views mathematics as a tool of man which enables him to understand and control his physical and economic environment, and as a language which makes mathematical communication possible. The approach would present mathematics in the context of its genesis, creators, evolution and contribution to the development and maintanance of societies.

Unit: The sum total of the classroom experiences in mathematics to be dealt with during the course of this study.

Debriefing: A non-directive investigation which places responsibility on the student to assess what he has learned.

Regular Treatment: The sum total of all classroom instructional procedures and strategies generally or ordinarily employed by the classroom teachers participating in this study.

Content Objectives: Objectives of instruction related to the acquisition of mathematical knowledge, concepts and skills, and mathematical processes such as making guesses, formulating abstractions, generalizations and problem solving techniques. In this study, process objectives are considered as a subset of content objectives as advocated by Parker and Rubin (1966, pp. 1-4; 44-45; 48).

IV. DELIMITATIONS

1. The study was delimited by involving two high schools, one grade level, two teachers and ninety-four students.

2. The study was delimited to the study of trigonometry as prescribed by the regular curriculum guide of the province of Alberta.

3. The study was delimited by the length of time spent in the classroom (3 to 4 weeks).

V. OUTLINE OF THE THESIS

The study is organized around six chapters. In Chapter II, the pertinent literature is reviewed. The purpose of Chapter III is to describe the cultural treatment and the manner in which the instructional materials were collected and prepared. The design and procedures of the instructional portion of the study are outlined in Chapter IV, as are the sources of data, testable hypotheses, research questions and analyses used. In Chapter V the results, interpretations and conclusions of the study are given. The summary and suggests for further research are presented in Chapter VI.

CHAPTER II

THE RELATED LITERATURE

I. INTRODUCTION

The purpose of this literature review is to describe the origin and evolution of the cultural approach to mathematics instruction and to identify its major characteristics. Included also is an indication of the outcomes of such an approach as suggested by its major proponents. Finally, various experiments and investigations of an educational nature which relate to the cultural approach are reviewed.

II. THE ORIGIN AND DEVELOPMENT OF THE CULTURAL APPROACH

The first significant "new mathematics" development project in the United States was headed by the late Max Beberman and commenced in 1952. Several other projects including SMSG (School Mathematics Study Group) were begun shortly thereafter. Opponents of the new programs appeared quickly. In 1958, Morris Kline wrote an article denouncing some of the basic directions which the new courses were taking. The article received wide distribution through "The Mathematics Teacher" (October, 1958). He suggested that the problems with the existing mathematics curricula

related not so much to outmoded content as it did to the manner in which the content was taught. He did, however, criticize such "modern" content as set theory, symbolic logic, groups and abstract algebra, which were in his opinion inappropriate for school curricula. He opposed the rigorous deductive logical presentation of ideas, which were, he believed, contrary to all historical experience of mathematical discovery. In addition, he argued against the excessive use of abstractions which were not properly grounded in concrete experience of the physical origin of mathematics. He saw the "new mathematics" as mathematics removed from its proper and historic role in the scheme of things, and he believed that as a consequence, the life and spirit of the subject would be lost.

Kline's ideas about the reforms in the mathematics curriculum of the secondary school were not arrived at quickly, for his professional career in mathematics had been devoted to the investigation and teaching of the role and influence of mathematics in the evolution of societies, both ancient and modern. His thinking can best be summarized by the following:

> Knowledge is a whole and mathematics is a part of that whole. Mathematics in every age has been part of the broad cultural movement of that age. We must relate the mathematics of history, science, art, music, literature, logic, as well as any other development the topic in hand permits. We should try as far as possible to organize our materials so that the development of the mathematics proper is related to the development of our civilization and culture. At the very least, each major topic should be imbedded in the

> cultural context which gave rise to it and should be capped by a discussion of what the creation has done to influence the development of our civilization. (Kline, 1958, p. 426)

But Kline was not the only mathematician in the United States concerned about the new courses. In 1962 seventy-five influential American mathematicians and educators, including Ahlfors, Courant, Coxeter, Pollak, Polya, Sawyer and Wittenberg, composed a statement expressing grave doubts about the direction the new programs had taken. The concerns expressed were similar to those outlined earlier by Kline. They insisted that the emphases given in the new programs were inappropriate for most students, separated mathematics and science even more, and isolated mathematics from its significant relation with other areas of human interest.

Though the major emphases of Kline's proposals are not directed at the body of content which makes up a mathematics curriculum, he nowhere minimizes the importance of content. His proposals relate primarily to the manner in which it is handled, for he stated:

> Mathematics is more than a method, an art, and a language. It is a body of knowledge with content that serves the physical and social scientist, the philosopher, the logician, and the artist; content that influences the doctrines of statesmen and theologians; content that satisfies the curiosity of the man who surveys the heavens and the man who muses on the sweetness of musical sounds; and content that has undeniably, if sometimes imperceptibly shaped the course of modern history. (Kline, 1957, p. 9)

Nearly two decades have passed since Kline's

criticisms and alternate proposals were first published. Since then, educators in Europe and North America including Athen, Weinberg, May, Higginson, Ailles, Norton, Steele, Hall, Freudenthal, Howson, Griffiths, Hill, Coleman and Edwards have advocated in varying degrees and with varying emphases, the teaching of mathematics at the secondary school level from a cultural perspective.

One of the central areas of concern has been the place and scope of mathematics in the general education of the student. Relative to this question, Kline (1973, p. 145) has suggested that the mathematics education of students should be broad rather than deep; a truly liberal arts education, where students not only get to know what the subject is about but also what role it plays in our culture and our society. Athen (1963, p. 240) advised that concrete knowledge in the various school subjects must be examined continuously with regard to its role in education. He states further that mathematics should be presented in a manner "which enables the young student to become conscious of his dependency, of his relations, and of his position in the world in which he lives." Athen (1964, p. 250) asserts that within the general aims of education in the secondary school, mathematics has a well explained function in the sense of one of the humanities which aids in interpreting the totality of our culture and civilization. Concerning the approach to mathematics education

May (1971, p. 100) believes that the subject should be taught as an integral and vital component of human culture, as an intellectual and scholarly discipline including theory and practice. He advocates a balanced view of mathematics which reveals all its various interrelations with other aspects of human activity. Weinberg (1965, p. 606) proposes a similar view:

> Elementary education should recapitulate the historic path of the discipline: its connections with other disciplines and with practical purposes, its origin, its scholarship—in short, its place in the scheme of things.

The thinking that mathematics should be approached from a broader perspective is coming increasingly into prominence. Howson (1973, p. 10) summarized the general tone and direction of the <u>Second International Congress on Mathematical Education</u> as follows: "There was a need to see mathematics not only as a world in itself, but as a part of a greater universe. How did mathematics evolve? What is its place in general education, indeed in civilization?" A recent report of the Science Council of Canada (Coleman et al., 1975), entitled "Mathematical Sciences in Canada" contains information and recommendations relative to secondary school mathematics education in Canada. The authors (Coleman, Edwards and Beltzner, 1975, p. 91) suggest that mathematics should be viewed (a) as a way of thinking which provides a powerful tool for analyzing subtle and unobvious aspects of experience, (b) as

a cultural resource which can add interest and enjoyment to life and (c) as a language which is essential for the communication of ideas for the formulation of societal goals. These broad aims for mathematics education are congruent to the basic goals and intents of the cultural approach.

One of the components of the cultural approach involves the inclusion of the history of mathematics as an integral part of instruction. Barzun (1945, p. 82) has stated:

> I have more than an impression—it amounts to a certainty—that algebra is made repellent by the unwillingness or inability of teachers to explain why . . . there is no sense of history behind the teaching, so the feeling is given that the whole system dropped down ready-made from the skies, to be used only by born jugglers.

Two decades later Hall (1973, p. xii) noted that students are seldom if ever provided with any significant amount of background or framework for the mathematics they have learned. He advised that teachers should "explain, by showing the actual historical stages of the metamorphosis, what mathematics is and why it has become the way it is." Leibniz is known to have felt:

> Nothing is more important than to see the sources of invention which are, in my opinion, more interesting than the inventions themselves. (Quoted in Polya, 1957, p. 123)

Schaaf (1963, p. 42) has stated "Probably no subject loses so much as mathematics when it is dissociated from its history."

Several possible outcomes are suggested as a result of utilizing the historical dimensions of the subject. Howson (1972, p. 39) suggests that the use of historical items reveals mathematics as a slowly but continuously evolving body of knowledge, a product of the creative spirit of man. In addition, he states, the evolving and refining of mathematical structure and symbolism becomes apparent. The Thirty-first Yearbook of the National Council of Teachers of Mathematics (1969, p. 13) states that a "sufficiently concrete and detailed tracing of the history of the development of a generalized idea is one of the best ways to teach an appreciation of the nature and role of generalization and abstraction." The Report of the Cambridge Conference on School Mathematics (1963, p. 19) pointed out two possible outcomes. The report suggested that providing the historical background of a topic often makes clear the motivation for discussing it as well as illuminating the essential truth that all known mathematics was invented by somebody. Kline (1956, p. 10) and Howson (1973, p. 39) have stated that the inclusion of the history of mathematics reveals the crucial role of mathematics in the development and maintenance of our modern technically oriented society.

The humanizing of mathematics is seen by several authors (Higginson, 1973, p. 143; Sobel and Maletsky, 1975, pp. 10,11) as a significant outcome of an emphasis

on the historical aspects of the discipline. Sarton (1936, pp. 8-28) has declared that a study of the history of mathematics will not make better mathematicians, but gentler ones. He states that the history of mathematics reveals it as a human creation about which all can be justly proud. It reveals also the inventive genius, and the curiosity which is uniquely characteristic of man.

According to Frietag and Frietag (1957, p. 222), an historical approach which emphasizes the origin, evolution and human aspects of mathematics takes care of the feelings as well as the mind of the learner, for they state "Connectionism stresses the need for establishing associational bonds in the learning situation. The historical method associates mathematical ideas with their creators, mathematics with the history of human affairs." In this regard, they suggest, the approach is in harmony with the Gestalt theory of learning which emphasizes the mental-emotional unity of the learner. In a similar vein, Whitehead has observed:

> You cannot put life into one schedule of general education unless you succeed in establishing its relation to some essential characteristic of all intelligent or emotional perception. (As quoted by Higginson, 1973, p. 137)

Another component of the cultural approach involves the role of applications of mathematics in the school mathematics curricula. Shortly after the "new mathematics"

programs were introduced into the schools, the Report of the Cambridge Conference on School Mathematics (1963, p. 21) made several recommendations concerning the future direction of mathematics education in the United States. Some of these recommendations were aimed at rectifying various curricular imbalances which were evidenced in a multitude of new textbook series and related classroom materials. One of the recommendations concerned the rationale for and the role and importance of "applications" in the modern mathematics curriculum. A decade later however, Ailles et al. (1973, p. 8), Bell (1974, p. 293) and Coleman et al. (1975, p. 103) observed that most secondary school mathematics programs placed little emphasis on the applications of mathematics, and especially to business and social problems.

Freudenthal (1973, p. 16) and Kline (1973, p. 148) point out that the main reason why mathematics continues to maintain a central position in modern school curricula is because it is so useful to man. This fact, they believe, should be reflected in the curricula. Freudenthal (1968, p. 5) has stated:

> Since mathematics has provided indispensable for the understanding and the technological control not only of the physical world but also of the social structure, we can no longer keep silent about teaching mathematics so as to be useful. In education philosophies of the past, mathematics often figures as the paragon of a disinterested science. No doubt it still is, but we can no longer afford to stress this point if this keeps our attention off the widespread use of mathematics and the fact that mathematics is needed not by

a few people, but virtually by everybody.

Bell (1974, p. 200) asserts that unless students of mathematics are able to solve problems which involve mathematical modelling of real world situations, mathematics will, for most of them, be pointless. Athen (1964, p. 244) points out that it is not possible to demonstrate the significance of mathematics in secondary schools without referring to its use in modern science, economy, and technology.

Most authors who advocate a greater stress on the usefulness of mathematics indicate that greater student interest and motivation follow. Cooney, Henderson and Davis (1975, p. 73) and Krutetski (1969, p. 118) contend that student interest in a topic increases when its utility has been perceived. Bassler and Kolb (1971, p. 25) state that "learning is facilitated if the learner perceives the task as being meaningful and applicable." Pollak (1970, p. 333) contends that unless the "applications" aspect of mathematics is presented as a vital part of the mathematics curriculum, students will not come to a full and honest picture of the subject, for he states "it is an important part of both the value and the heritage of mathematics to see and practice its usefulness."

Hill et al. (1975, p. 138) reiterated the plea for teachers to provide students with a greater opportunity to apply mathematics in as wide a realm as possible—in the

social and natural sciences, in consumer and career related areas as well as in any real life problems that can be subjected to mathematical analysis.

In order to reveal mathematics not only as a discipline which possesses characteristics unique to itself, but also as a discipline which interrelates with other areas of human interest, Hawkins (1972, p. 120) insists on closure of the mathematical domain. By closure he means the mathematical use of the word, which implies removing barriers, not building them. He suggests that the mathematical domain be defined in such a way that it does not exclude any situation of learning merely on the ground that the latter might also be described under social, scientific or aesthetic categories. Similar views have been expressed by Reichmann (1967, p. 256) and Edwards (1973, p. 3). Matthews (1972, p. 78) has noted that the first reforms in mathematics were made within the subject, but that the second reforms will involve looking across the boundaries. He has stated "We have had a glimpse of the way ahead—mathematics with everything."

A multi-related approach to mathematics establishes "outside connections" with the subject matter. "New mathematics" is connected within the subject, a connection which is constructed in order to teach a unified mathematics. From the standpoint of the student however, Freudenthal (1973, pp. 74-76) sees little value in this

type of connection. He suggests that there are other more important connections which should be tapped:

> To teach connected mathematics it is not wise to start looking for direct connections; they should rather be found between the contact points where mathematics is attached to the lived-through reality of the learner. Reality is the framework to which mathematics attaches itself, and though these are seemingly unrelated elements of mathematics, in due process of maturation connections will develop. Let the mathematicians enjoy the free-wheeling system of mathematics—for the non-mathematician the relations with the lived-through reality are incomparably more momentous.

Another component of the cultural approach to mathematics instruction is the importance attached to the pursuit of affective goals. Mathematics educators generally divide the goals of mathematics education into three categories: content goals, process goals and affective goals. The emphasis given particular goals is determined to a greater extent by the biases and philosophical view of mathematics held by the teachers and the curriculum designers. The Mathematics Curriculum Guide (1971) of the Alberta Department of Education lists as one of the five major objectives for the senior high school mathematics program "To develop an appreciation of the contribution of mathematics to the progress of civilization" (p. 3). It is probably fair to assume however, that curriculum designers and teachers have made little attempt to fulfil the intents of this objective.

Bassler and Kolb (1971, p. 45) have observed that teachers feel guilt when they turn their attention from

meeting content objectives for any length of time. They contend that "the importance of affective goals should be more widely recognized and stressed so that their achievement can be pursued without apology." Concerning the teaching of mathematics in the secondary schools of Ontario, Ailles, Norton and Steele (1973, p. 3) stated that "something needed to be done to make the study of mathematics a more meaningful and even exciting experience for a greater number of students." Sawyer (1973, p. 10) has suggested that the main task of any teacher is to make the subject interesting. This is an important observation since as Cooney, Henderson and Davis (1975, p. 70) point out, attitudes toward mathematics are learned concomitantly with the subject matter. When students find the subject matter interesting and relevant, they suggest, their motives for studying increase and they are more likely to retain it. Johnson (1957, p. 116) argues that:

> Since we learn according to our reactions to experiences, we cannot expect an attitude of appreciation to emerge from classroom lessons that are dull and uninspiring or from homework that is meaningless drudgery.

Two national reports on mathematics education (Hill et al., 1975, p. 142; Coleman et al., 1975, p. 62) dwell at length on the importance of affective goals in the teaching-learning environment. Hill et al. (1975, p. 142) recommended "that the affective as well as cognitive domains in mathematics should be the subject of

constant and programmatic attention." A renewed interest in affective goals is also evidenced by recent recommendations that research be carried out to investigate more carefully the relationship between the cognitive and affective domains (Hill et al., 1973, p. 144; Griffiths and Howson, 1974, pp. 48-49).

Most of the outcomes associated with the cultural approach to mathematics instruction relate directly or indirectly to the achievement of affective goals. Kline (1962, p. viii) states that most students will respond to the cultural approach because it shows that mathematics has a point. He believes that because students have failed to see the full significance of mathematics they tend to dislike it, do poorly in it, depreciate its value, and shrink from further involvement. He suggests however that "if we do succeed in interesting students in our subject, we may get them to appreciate its value as a discipline, an art, and an engaging intellectual activity."

Concerning the "appreciation of mathematics," different individuals find fascination and satisfaction in different aspects of the discipline. Jon von Neumann (1961, pp. 1-9) and Wigner (1969, p. 124) wondered at the peculiar and inexplicable relationship between mathematics and the natural sciences. They viewed it as something bordering on the mysterious, for which there was no rational explanation. But appreciation of the charm,

beauty and power of mathematics can come from sources other than perceiving its utility in the ordering of man's world. Fadiman admits to being mathematically illiterate, but as a result of his reading about and around the subject he has stated, concerning the fascination of mathematics:

> I have sensed that charm as vividly as one may sense the charm, without ever being quite able to define it, of a lovely face or voice or a piece of architecture . . . (Quoted by Butler and Wren, 1960, p. 117)

Maslow (1971, p. 178) confesses that mathematics can be just as beautiful and just as peak-producing as music. He, like Fadiman, failed to discover the aesthetic pleasure derivable from mathematics until later in life. They both attribute this to a "bad case of early pedagogy." The "appreciation" of mathematics as mentioned by Fadiman and Maslow, neither of whom are mathematicians, underlines the following observation of Resnikoff and Wells (1973, p. 3). They have noted that just as it is possible to appreciate a piece of music without being a composer, it is similarly possible to appreciate the diversity, utility and beauty of mathematics without being a mathematician.

By presenting mathematics from a cultural perspective and revealing it as the Queen as well as the handmaiden of the sciences, of having both aesthetic and utilitarian attractions, advocates of the cultural approach suggest that due consideration is given to each of the three major goals of mathematics instruction: content goals, process

goals and affective goals. (In this study process goals are considered to be a subset of content goals.)

III. ATTEMPTS AT IMPLEMENTATION

Over the years attempts have been made to implement various facets of the cultural approach into regular classroom instruction. During the fifties, for example, a great deal of interest was expressed in the history of mathematics as an essential component of mathematics instruction and as a useful pedagogical tool. This emphasis was evidenced by the proliferation of articles in the professional teaching journals, extolling the advantages of such an approach. Jones (1957, p. 59) observed however, that in spite of these and earlier efforts to promote the historical bias, little impact had been felt at the classroom level.

In 1969 the annual yearbook of the National Council of Teachers of Mathematics was entitled "Historical Topics for the Mathematics Classroom." The primary aim of the book was to "make available to mathematics classes important material from the history and development of mathematics, with the hope that this will increase the interest of the student in mathematics and their appreciation for the cultural aspects of the subject" (p. x). It is not possible to assess the impact of this volume on actual classroom practice. New interest in finding a place for

history in the mathematics curricula has been pointed out by Howson (1972, p. 39) who also notes that its universal neglect has been the cause of much concern. Cooney, Henderson and Davis (1975, p. 116) suggest that teachers probably do not use the historical approach frequently because they do not know the history of mathematics as well as they know mathematics.

In 1972, St. Paul's High School in New Hampshire conducted an experimental course on "The History of Mathematics" (Dittrich, 1973). The course was a half-year elective for eleventh and twelfth-grade students and was used to complement and enrich the mathematics curriculum. The course dealt with such philosophical questions as "What is mathematics?", "What is logic?", "What is the role of logic in mathematics?" and involved extensive reading in mathematics literature in and around the discipline. Other topics ranged from discussions concerning the Peano postulates to the mathematics of the Parthenon. In spite of the general nature of the course, Dittrich (1973, p. 37) found no discernible downward trend in the Stanford Achievement Test (SAT) scores of those in the course. He also reports that students repeatedly confirmed that the course was the most appealing they had ever taken in the subject. The course is now offered regularly.

The introduction of applications of mathematics into the general mathematics curricula of the high schools

in North America has been slow. Concerning this matter Pollak (1973, p. 334) observed, "no one seriously questions the importance of applications of mathematics and yet we have had (and continue to have) enormous difficulty in obtaining a proper place for them in the curriculum." In the United States during the past decade, much effort has been given to develop materials which were "applications oriented." The earlier efforts were directed toward providing supplementary materials for existing programs which would provide appealing motivational and illustrative applications. The impact of these materials, produced primarily by SMSG (Bell, 1967, 1973) is difficult to assess.

Bell (1971, pp. 293-300) carried out a study with one class of first-year algebra students at the University of Chicago Laboratory High School. The purpose of the study was to determine whether contemporary uses of mathematics and mathematical modelling could be made a useful part of a standard school mathematics course. Bell found, in this pilot study, that it was possible to give consideration to the applications of mathematics within the standard first-year course and that this emphasis can work in harmony with the subject-matter demands of the course. He found also "that there is tentative but encouraging indication that these efforts resulted both in more awareness of the usefulness of mathematics and in

positive attitude changes" (p. 300).

There are at least five current curriculum development projects illustrating major new ways to blend mathematics and applications at the secondary level. Included among these are the UICSM "Introduction to Mathematical Methods in Algebra, Geometry, and Probability," Usiskin's "First Year Algebra Via Applications" and "The Man Made World." Of these mathematics/applications curricula, only one, the "The Man Made World," has been completed and field tested. Hill et al. (1975, p. 30) suggest "that both the extensive claims for the meaningfulness of applications in the curriculum and the volume of activity taking place to develop them point to the necessity of early and serious evaluative efforts."

Following the aims of education in Alberta as perceived by the Worth Report (Worth, 1972), Higginson (1973, pp. 137-153) has advocated a mathematics curriculum centred about the aesthetic, occupational, social, intellectual and recreational needs of the students. He argues that traditional mathematics curricula have been almost completely antithetical to the achievement of person-centred goals, even though, he suggests, mathematics is potentially one of the most potent vehicles for the achievement of these goals. He has not however, developed instructional materials which would give evidence of how this might be carried out in the classroom environment.

One of the most extensive curricular reform projects in the United States was "Harvard Project Physics" (Rutherford, Holton and Watson, 1970). A major goal of the project was to design a humanistically oriented physics course. The materials which were developed, over a period of eight years, consist of student readers, student handbooks and texts. The reader includes interesting historical notes, brief biographies of physicists, and short articles on the use and importance of physics in modern society. This effort to portray the spirit as well as the nature of physics is characteristic of the texts as well. Concerning the success of this series the authors report:

> The large-scale study of student achievement and student opinion in the participating schools throughout the United States and Canada showed gratifying results—ranging from excellent scores on the College Entrance Examination Board Achievement Test in Physics to the personal satisfaction of individual students. (p. x)

This unique curriculum series in high school physics, which has no counterpart in mathematics, characterizes better than any other program to date, the spirit and intent of the cultural approach as outlined in this study.

IV. SUMMARY

The literature review has outlined the genesis, evolution, major characteristics and proposed outcomes of an instructional approach to mathematics which would

present the subject as the multi-dimensional discipline that it is.

The method would reveal mathematics as an evolving creation of the human intellect and would emphasize the fact that it is man's most vital tool for the proper understanding of his physical, social and economic worlds. In addition, mathematics would be viewed as a language which enables man to communicate the realities and relationships between the physical and mathematical world. The approach would present mathematics as a body of knowledge, of techniques, skills and structures, knowledge of which enables man to mathematize reality and perhaps even that of mathematics. The multitudinal interrelations of mathematics with other domains of human interest would be emphasized as well.

But as man is more than the sum of his physical components, so too is mathematics more than the sum of its parts. The cultural approach would incorporate the cultural heritage of mathematics as part of the regular teaching strategy. Whenever possible, the content of a topic would be presented in the context of its historical origin and the evolution of the topic would be traced through time. The impact and significance of the invention on the development of mathematics as well as on the culture of the time would be investigated. In addition, the importance and role (of the concept or topic under study)

to modern society would be made apparent through problems appropriately chosen to demonstrate the utility of the topic.

As a consequence of this approach, proponents claim that a truer, more balanced and honest view of mathematics would result. Students would come to appreciate mathematics as an artistic product of man, but one which, paradoxically, provides him with his greatest tool for understanding and controlling his environment. The significance and vital role of mathematics in the efficient operation of the modern technological society of the student would be apparent. In addition, advocates claim, students would come to appreciate more thoroughly the beauty, power, utility and spirit of mathematics.

Kline (1967, p. 9) has stated:

> We should like to understand what mathematics is, how it functions, what it accomplishes for the world, and what it has to offer in itself.

The aim is to teach mathematics in context, so that students might come to sense the flavour as well as the ingredients of mathematics, to see mathematics as Fadiman (1957, p. 8) has seen it "as a kind of hub of the universe from which radiate the spokes of a hundred arts and sciences . . . or like a circular window opening on 360° of thought"—in short—not as a subject but as a world.

> The humanization of mathematical teaching, the bringing of the matter and the spirit of mathematics

to bear not merely upon certain fragmentary faculties of the mind, that this is the greatest desideratum is, I assume, beyond dispute. (Keyser, 1940, p. 61)

CHAPTER III

THE CULTURAL MODEL

I. INTRODUCTION

Proponents of the cultural approach to mathematics instruction propose that the subject should be taught in a manner that reveals its multiple dimensions as an art, a language, a process, a body of knowledge, and as a tool. They also propose that instruction in mathematics should reveal its rich history, its development, its creators, its contribution to the aesthetic and utilitarian needs of man, and as a discipline which has affected and continues to affect the course of history and the makeup of cultures.

The purpose of this chapter is to state the characteristics of a cultural model for mathematics curriculum design based on the literature, to describe the cultural treatment, and to indicate the manner in which instructional materials for a unit of Mathematics 23 trigonometry were collected and prepared. The chapter concludes with a description of the regular treatment.

II. SELECTION OF CONTENT AREA

The content area selected was elementary trigonometry for students in Mathematics 23 classes, as prescribed by the Alberta Department of Education Curriculum Guide to Mathematics (1971). Trigonometry was selected because it

required mastery of minimal prerequisite mathematics, because resource materials were available, and because the time required to complete the unit (four weeks), seemed adequate for the in-school instructional time required for a study of this sort.

The prescribed content of the Mathematics 23 unit on trigonometry as found in the Department of Education Curriculum Guide is as follows: review; the six basic trigonometric functions; special and quadrantal angles, and graphs.

III. THE CULTURAL MODEL

The cultural approach to curriculum design acknowledges the importance of the content, process and affective goals of mathematics instruction (in this study, process goals have been earlier defined as a subset of content goals). In the cultural approach, considerable effort is given to the achievement of affective goals, but not, ideally, at the expense of the achievement of content goals. Concerning affective goals, Bassler and Kolb (1971, p. 44) suggest that the study of mathematics and the exposure to mathematical activity should result in valuing

> 1. The ability of human intelligence to invent and discover relationships whose application permits man to influence and order his environment.
> 2. The ability of human intelligence to go beyond the known and observable part of its physical environment and engage in imaginative thinking.
> 3. The enjoyment that can result from intellectual pursuit and a love of knowledge.
> 4. Mathematics and mathematical activity as a substantial part of the cultural heritage of the human race that deserves the support and encouragement of society.

The cultural approach is an attempt to present mathematics in such a manner that the achievement of both content and affective objectives are pursued with equal determination. Figure 1 illustrates the various areas of interest which are brought to bear on or connected with, the required mathematical content of the trigonometry unit.

In the cultural approach, the mathematical content of a unit of mathematics is complemented whenever and wherever possible with appropriate historical, applicational and other relevant information in an attempt to enrich, clarify and add a contextual framework to the learning process. An effort is made to fit the topic into its rightful place within the totality of human knowledge and history. The criterion for the selection of relevant materials is the one advocated by Edwards (1973, p. 9): "anything is relevant as long as it is connected and interesting."

Lessons are designed so as to embed the required content within the context of its historical genesis and development. The names, occupations, cultural environment and mathematical contributions of the creators of pivotal ideas (in this case trigonometric) are discussed, as well as the impact that their inventions had on the development of mathematics in general and on the society from which it arose in particular. Major breakthroughs in the evolution of the discipline are highlighted as they appear within the natural sequence of the course. In order to appreciate the extent to which trigonometric methods have

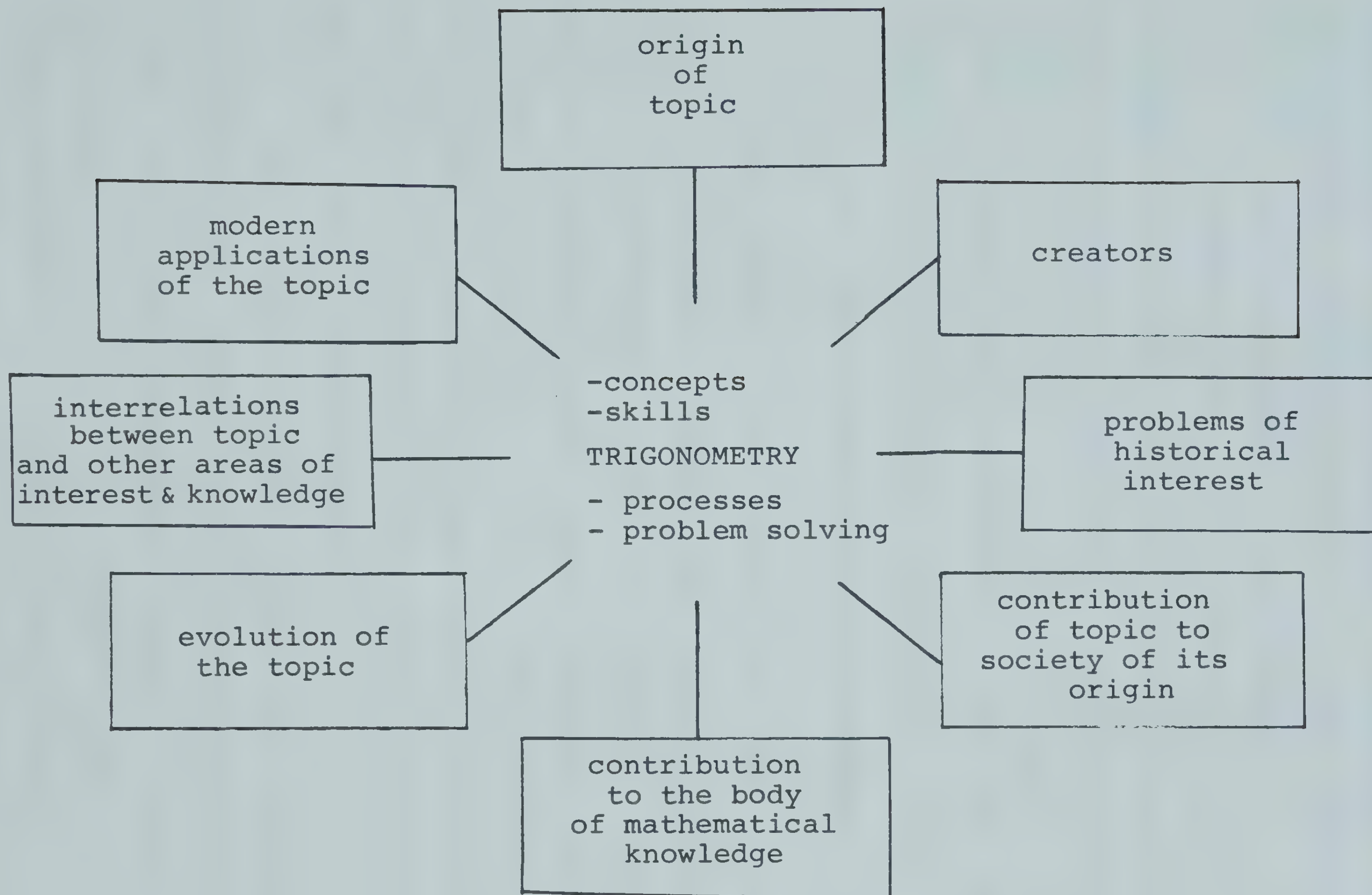

FIGURE 1

THE CULTURAL MODEL

been and are being utilized to solve problems, students are required to apply trigonometric techniques to problems of historical interest as well as to those originating in the real-world environment of the learner.

In summary, the cultural approach to instruction is an attempt to achieve both the content and the affective goals of mathematics education by presenting the topic under study in the context of its historical genesis and evolution, its role in the development of societies, and its contribution to the utilitarian and aesthetic needs of man.

IV. DESCRIPTION OF CULTURAL TREATMENT

The purpose of this section is to explain the cultural treatment as devised for the Mathematics 23 unit on trigonometry, and should be read in conjunction with the associated lesson support materials which are included in Appendices 1, 2 and 3. The design and function of the support materials (lesson guides, worksheets, historical and biographical summaries and projectuals) are outlined later in this chapter.

The treatment was built around and integrated with the mathematical content of the regular program of instruction. All of the trigonometry ordinarily covered in the Mathematics 23 classes in the schools in which the instruction took place was included in the treatment.

Trigonometry as we know it began in the Greek city of Alexandria about 150 B.C. Students were made aware of

the geographic location of Alexandria, the stature of the city in the Greek world of the time, and its important place in the development of trigonometry. The teacher discussed (with the aid of projectuals) the occupation and contributions of the ancient astronomers Hipparchus and Ptolemy—indicating that trigonometry had its beginning as a result of the desire and need of astronomers to understand and chart the heavens. Occasionally, students were shown or were asked to solve problems of historical significance. This required students to apply their knowledge of trigonometric techniques. One such problem involved investigating how Hipparchus (150 B.C.) was able to calculate the distance from the earth to the moon (Lessons 3 and 4; Worksheet 5 and 6). Another related to the mathematician-engineer Heron (100 A.D.), who used the methods of trigonometry to show his peers how a tunnel could be constructed through a mountain by starting at either side (Worksheet 5 and 6). By solving appropriate historical problems or from teacher-directed classroom discussions, students were informed of the contribution made to man's knowledge as a result of the invention of trigonometry. Students were told that as a direct consequence of the development of trigonometry, man was able to chart the heavens, devise a world grid system using longitude and latitude, and solve a host of problems in engineering, mathematics and navigation.

Concerning the sequence of topics or subtopics within a unit of high school mathematics, Kline (1973, p. 114) proposed that, as a general rule, the sequencing should parallel that which was traced out by the natural historic evolution of the topic. This approach he called the "Genetic Principle." The manner in which the unit in trigonometry was sequenced in the Mathematics 23 course followed Kline's Genetic Principle. Thus it was possible to indicate significant developments in the natural evolution of trigonometry as the unit progressed. Students were informed of the construction and development of trigonometric tables, and were made aware of the amount of time required to calculate them. A comparison was made between the time required by Rheticus (1550 A.D.) to calculate tables and the time required to do similar calculations today using modern computing devices (Lesson 2 and Projectual 8). Prior to the drawing of trigonometric graphs, the significance of the work of Viéta and Déscartes was discussed as well as how these advances affected the development of trigonometry (Lesson 9).

At the beginning of the unit of instruction, teachers of both the cultural and regular groups pointed out the importance of trigonometry as a useful tool of modern man. In particular, mention was made of its central importance in indirect measurement and in providing a mathematical model for the proper description and understanding of

periodic events. Throughout the unit an attempt was made to provide example and assignment problems which reflected the diverse manner in which trigonometry finds application in the modern world. Problems were constructed using data collected from various sources in and around the city of Edmonton (Worksheets 3 and 4; Worksheets 5 and 6).

Both the cultural and the regular classes spent two periods studying the mathematics and use of the clinometer in indirect measurement. One of the two periods was spent outside in order to demonstrate the usefulness of the device as an aid in calculating the heights of various objects in the vicinity of the school.

Concerning the manner in which the mathematical content of the unit was presented, the teachers were encouraged to maintain their regular classroom style of presentation. This was done in order to maintain, as much as possible, the normal classroom environment. The lesson outlines given the teachers were descriptive rather than prescriptive, particularly as they related to the manner in which specific trigonometric concepts or generalizations were to be presented. The amount of homework assigned was minimal, but decisions concerning the amount and the checking of homework assignments was left to the discretion of the instructors.

V. THE PREPARATION OF INSTRUCTIONAL MATERIALS

Content Scope and Preparation of Objectives

Before the lessons and support materials were constructed, the researcher met with the participating teachers in order to determine the content material which could be adequately studied in four weeks of class-time. As a result of this meeting, the trigonometric content was further delineated and the content objectives specified. Both teachers agreed that the content objectives identified learning outcomes which students of Mathematics 23 would ordinarily be expected to achieve within a four week instructional period.

The specified content objectives were common to both the cultural and regular groups, but certain affective objectives were identified for the cultural group only. Lesson objectives were written at the top of each lesson guide and affective objectives identified with an asterisk. For a listing of the objectives see lesson guides contained in Appendix 3.

Sources of Materials

In order to provide for an appropriate treatment, materials were assembled from various sources. Most of the information and materials were collected from mathematics books written from a cultural or historical perspective. Other materials were taken from books written

about mathematics for the popular market. Some of the books used are mentioned in the following section. For a more complete listing see the entries in the Bibliography marked with an asterisk.

Historical and Biographical Sources

Since the lives and contributions of mathematicians and the historical development of mathematics played an important role in the cultural treatment, books on the history of mathematics such as Eves' (1965) "An Introduction to the History of Mathematics" were consulted. This book also contains concise biographical sketches of mathematicians as does "Historical Topics for the Mathematics Classroom" (1969). Other books such as "The City of the Stargazers" (Huer, 1972) provided more specific information about the Greek city of Alexandria. From these same sources came several problems which were of historical interest and whose solutions were within the capabilities of the students.

Cultural Sources

Several books such as "Mathematics: A Cultural Approach" (Kline, 1962), provided information on the interplay between the creators of trigonometry, the mathematics created, the impact of the inventions on the society from which it grew, the major developments of the topic over time and the contribution of the topic to modern society.

Content and Applications

Many of the exercises and problems provided on the student worksheets came from high school mathematics textbooks, including the one used regularly by students in Mathematics 23. Since one of the aims of the cultural treatment was to show the extent to which mathematics finds application in the world, problems illustrating its usefulness were made an integral part of the classroom routine. Appropriate problems were found in some school mathematics textbooks, SMSG supplements (Bell, 1967, 1972) or constructed by the researcher. Information for the construction of problems was obtained by writing or telephoning Wardair, Alberta Government Telephones, Alberta Department of Highways and Banff National Park. Materials incorporating this information appear in Appendix 3.

Support Materials

1. Historical and Biographical Summaries

When sufficient appropriate and relevant historical and biographical materials were assembled, summaries were written, highlighting the life, times, and contributions of the mathematicians responsible for significant breakthroughs in trigonometry. These summaries were provided to each teacher as an aid to preparation for the portions of the cultural treatment which required an historical discourse. Copies of these summaries are included in Appendix 1.

2. <u>Summary of the Uses of Trigonometry</u>

Trigonometry has gained for itself a permanent place in the world of mathematics because of its power as a tool for mensuration and as a mathematical model for situations that are periodic. A summary was devised which indicated the diverse ways that trigonometry finds application today in each of the two major areas (mensuration and periodicity). The materials contained in Appendix 1 were taken from a teaching module on trigonometry prepared by Ellis (1976).

3. <u>Projectuals</u>

Several projectuals were prepared for use in the treatment classes. Projectuals were made from maps which were intended to give geographical perspective to historical topics, and from drawings of mathematicians. Often the projectuals displayed brief historical or biographical information. Historical problems of interest and problems of a more general nature were also made into projectuals. The projectuals, as well as adding a visual dimension to the presentation also provided the teacher with a visual aid, particularly when the display summarized the life and contributions of particular mathematicians. Masters of the projectuals are displayed in Appendix 2.

4. <u>Worksheets</u>

Student worksheets were devised containing exercises and problems, and were prepared in order to provide

in-class and homework assignments. On occasion, the worksheets for the cultural and regular groups were identical. Often however, the problem sets were outwardly different but were equivalent from the point of view of the mathematical model required to solve them. Copies of the worksheets can be found in Appendix 3.

5. Lesson Guides

Lesson guides were provided for each of the instructors involved in the study. They contained a statement of the general purpose of the lesson, a list of lesson objectives, and a lesson outline which proposed a feasible sequence of instructional displays. In addition the lesson outlines indicated the resource materials to be used in conjunction with the lesson. Each instructor received a three-ring binder containing the lesson guides, student worksheets, historical and biographical summaries and projectuals. These materials are contained in Appendices 1, 2 and 3.

6. Achievement Tests

Tests, designed jointly by the researcher and the teachers involved, provided a fair and adequate coverage of the mathematical content studied prior to the test. Test questions tested content objectives only and were similar to problems completed in class. Two achievement tests were administered to each class during the three to four week period of instruction, the first midway through

the unit, and the second at the end of the unit. The second achievement test covered the mathematics studied over the entire unit, but with emphasis given to the mathematical topics studied during the last two weeks of class.

The affective objectives and related outcomes of the instructional period were tested at the conclusion of the unit.

VI. DESCRIPTION OF THE REGULAR TREATMENT

The purpose of this section is to describe the regular treatment. The mathematical content studied was identical to that studied under the cultural treatment. The classes were fifty minutes long and met four times per week, as did all classes participating in the study. The content objectives were identified at the top of the lesson guides. Worksheets were made available to students in the regular classes as well.

In a given school, the regular treatment was defined as the sum total of all classroom instructional strategies and procedures ordinarily employed by the participating classroom teacher. A typical lesson consisted of taking up the previous day's assignment, teaching a new trigonometric concept (employing a "question and answer" approach), solving example problems on the blackboard, and making a related assignment to be worked on in class. The instructor circulated throughout the class during this

work-session. The homework demands were minimal. For further clarification of lesson sequence and content, refer to the lesson guides in Appendix 3.

VII. SUMMARY

The purpose of Chapter II was to identify, through a review of the literature, the major characteristics of a cultural approach to mathematics instruction. The purpose of this chapter was to indicate the manner in which instructional materials for a unit of grade eleven trigonometry were collected and assembled using as a basis for design, the "cultural model." The supporting materials identified and described included historical and biographical summaries, lesson guides and projectuals for teacher use, and tests and worksheets for student use.

The cultural treatment described was designed to complement and supplement the required mathematical content of the unit. Throughout the unit, background information of an historical or biographical nature was introduced into the instructional display. The varied applications of trigonometry were indicated by showing through assigned problems and classroom examples how the methods and techniques of trigonometry have served the varied requirements of man through the centuries. The treatment was also designed to portray the fact that trigonometry has evolved as a result of a unique blend of the needs, inventive

genius, and curiosity of man.

CHAPTER IV

DESIGN AND RESEARCH PROCEDURES

The purpose of this study was to design and produce classroom instructional materials for a unit of grade eleven high school trigonometry from a cultural perspective, to teach the unit in a school setting and to seek answers to questions concerning resultant cognitive and affective outcomes. The purpose of this chapter is to specify the nature of the sample, the measures used, the testable hypotheses, the research questions and to indicate the analyses used.

Throughout this study the participating teachers were actively involved in decision making pertaining to the instruction. Teachers had input into decisions concerning the scope of the content studied and the design of tests. Teachers graded their own test papers according to their own marking scheme and were encouraged to maintain as much as possible their regular classroom teaching style. This was done in order to preserve the usual classroom atmosphere. The approach to the evaluation of the resultant instructional outcomes was the same as that suggested by Bell (1971, p. 300), who stated, concerning a pilot study which had characteristics similar to this one: "controlled research studies were not appropriate in

the pilot study . . . but some evaluative efforts were carried through in the spirit of gaining experience and insights that might be helpful in guiding future efforts."

I. THE SAMPLE

The sample consisted of ninety-four students from four coeducational classes of grade eleven (Mathematics 23) students, two classes from each of two high schools under the jurisdiction of the Edmonton Separate School Board. The researcher requested, of the Board, that it attempt to provide schools which had at least two Mathematics 23 classes in session and where one teacher taught at least two of these classes. Schools were consequently assigned which met the conditions requested. Only two teachers therefore, were involved in the study.

II. ASSIGNMENT OF CLASSES TO TREATMENTS

In each of the schools participating, one class was taught the unit in the manner usually taught by that teacher and the other class was taught the unit from a cultural viewpoint. Classes were assigned to treatments using the flip of a coin.

If the two schools are represented by the letters A and B, and if the two treatments are represented by R (regular) and C (cultural), let AC, for example, represent the class in school A under the cultural treatment.

Hence the classes can be represented by AR, AC, BR and BC. Table 1 indicates the number of students in each of the classes.

TABLE 1

NUMBER OF STUDENTS IN PARTICIPATING CLASSES

School	Treatment		Total
	R	C	
A	28	28	56
B	12	26	38
Total	40	54	94

The duration of the in-school portion of the study differed between the two schools. The time made available for instruction at school A was three weeks whereas the time allotted in school B was four weeks. As a consequence less mathematical content was studied in school A than in school B. Except for the time differential, the two school environments provided opportunity to test the cultural model under similar conditions.

III. PREPARATION OF TEACHERS

Prior to the in-school instructional period, the researcher met individually with the teachers to explain the purpose of the study and the nature of the cultural treatment. Later, a joint meeting was held at which time

final decisions were made concerning the duration of the instructional period, scope of the trigonometry content to be studied, and the testing and grading procedures. In addition, the characteristics and proposed outcomes of the cultural treatment were further discussed and clarified.

During the three to four weeks of instruction, the writer visited about half of the regular and treatment classes. This provided the occasion for ongoing inservice through a discussion of the objectives, instructional methods and aids available for particular lessons.

IV. SOURCES OF DATA

Data used in this study came from the school officials, student achievement scores on the unit, student reaction questionnaires and teacher reaction questionnaires.

Achievement Scores in the Regular Mathematics Program

The achievement grade in the regular mathematics program was obtained for each student. These scores, which were obtained from the schools, were the scores received in Mathematics 23 and reported on the last report card prior to the commencement of the instructional period.

Achievement Scores on the Trigonometry Unit

Each student took two achievement tests in trigonometry, the first at the midpoint of the unit and the

second at the end. The final achievement score for each student was calculated by finding the mean of the scores obtained on the two tests. In a given school both the regular and treatment classes wrote the same test. All tests were graded by the participating teachers in accordance with their own marking scheme. This was done in order to maintain the approach to testing and evaluation which had been established in the classroom over the months prior to the instructional period.

By considering the two achievement tests as one, a measure of reliability was calculated using the "split-half method" as outlined by Ferguson (1971, pp. 366-367). The "split-half" scores required for the calculation were obtained by adding the scores received on the odd-numbered items from both tests and adding the scores received on the even numbered items from both tests. This was done for each student participating in the study. The correlation between these scores was calculated to be 0.81 and the resultant reliability coefficient over the two tests was 0.90.

Student Reaction Questionnaire

At the conclusion of the instructional period all students participating in the study completed a two-part questionnaire. Part 1 consisted of sixteen questions with Likert-type responses. Each of the sixteen items could be classified under one of four major areas. The

areas tested by the items related to the "difficulty" of the trigonometry studied, the student's "interest" in the trigonometric unit, the "usefulness" of trigonometry, and the "cultural relevance" of the topic. Appendix 4 contains a copy of the student reaction questionnaire as well as a listing of the test items grouped into the four areas of categorization.

The data from this part of the questionnaire were used to compare the responses of students under the two treatments with respect to the four areas. Part 2 of the questionnaire required students to respond to the following questions: 1. What did you like best about this unit of work on trigonometry? 2. What did you especially dislike about this unit on trigonometry? 3. What did you learn while studying this unit on trigonometry that surprised you most? In addition, students were encouraged to comment further on any aspect of their recent classroom experience. These responses too were used for the purpose of comparing student reaction to the regular and cultural treatments.

Debriefing

Near the end of the instructional period but before the last test was written, a random sample of students from each of the four classes was interviewed by the researcher. Each student was interviewed individually and asked to respond to each of the following three questions:

1. What have you learned as a result of studying this unit on trigonometry?
2. What do you know now that you did not know before studying this unit on trigonometry?
3. What can you do now that you could not do before you studied this unit on trigonometry?

Question 2 and 3 were essentially restatements of the first question. They were included to avoid any confusion over wording or interpretation of the word "learned." The students were allowed to take as much time as they desired to answer. The questions were related one at a time. Student answers were recorded on tape.

The purpose of the "debriefing" was to place the responsibility on the student to identify those facets or features of the trigonometry unit that were sufficiently important or interesting to warrant recall, i.e., to determine what the student felt was worth remembering. The responses were later classified under thirteen headings, ten of which represented specific subtopics of the unit and the remaining three were more general categories. Each unique response was subjectively "weighted," that is, assigned a numeral, representing its "complexity" as perceived by the researcher. A simple response such as "I can do problems involving right triangles" was assigned a weight of one. A response such as "If I know the height of a lighthouse above sea level, the angle of

depression from the top of the lighthouse to a ship at sea, then I can find the distance from the ship to the base of the lighthouse" was assigned a weight of two. An elaborate response indicating exceptional understanding (or several examples) was recorded as a weight of three. The data received from the debriefing were used to compare the type and complexity of the learning outcomes recalled under the two separate treatments.

Teacher Reaction Questionnaire

A week after the in-class instructional period concluded, the teachers involved in the study completed a questionnaire. The purpose of the questionnaire was to obtain teacher reaction to the unit and to obtain their assessments of the appropriateness and effectiveness of the treatment. A copy of the teacher questionnaire is contained in Appendix 4.

V. HYPOTHESES AND RESEARCH QUESTIONS

The purpose of this study was to investigate the efficacy of a culturally based approach to trigonometry instruction with respect to its effect on student achievement, certain affective outcomes and student recall of associated knowledge concerning the unit. The purpose of this section is to indicate the research questions posed, the testable hypotheses and the analyses used in the study.

QUESTION 1

Are there differences in achievement between students under the cultural treatment and students under the regular treatment?

This question can be stated equivalently in the following manner:

Hypothesis 1

There is no significant difference between the mean achievement scores on the unit between the regular and cultural classes within each school.

The students involved in the study were not assigned to classes randomly, nor were the classes chosen randomly, consequently, any difference in the dependent variable (achievement on the trigonometry unit) between the regular and cultural classes could be attributed to initial differences in their mathematical abilities. In order to statistically adjust for the possible effect of this uncontrolled variable (last report grade), the hypothesis was tested using analysis of covariance of the final achievement scores with the last report grade on the regular mathematics program as the covariate (Ferguson, 1971, pp. 288-299).

QUESTION 2

Are there differences in certain affective outcomes between students under the cultural treatment and students under the regular treatment?

Since the questionnaire items subdivide into four areas, this question can be stated equivalently using the following four hypotheses:

Hypotheses 2(a), 2(b), 2(c) and 2(d)

There are no significant differences between the questionnaire response scores of students in the cultural classes and students in the regular classes within each school with respect to the (a) difficulty items (b) usefulness items (c) interest items (d) cultural relevance items.

As discussed previously in this chapter each of the sixteen items of the questionnaire fell into one of four categories: difficulty, usefulness, interest and cultural relevance. Hypotheses 2(a), 2(b), 2(c) and 2(d) were tested using the chi-square test for independence (Ferguson, 1971, p. 188). Chi-square tests were made on the frequency of "favorable" and "unfavorable" responses between the regular and cultural classes. Given a five-choice Likert-type scale, a response was considered "unfavorable" if SD (strongly disagree) or D (disagree) were chosen and "favorable" if A (agree) or SA (strongly agree) were chosen (this general rule required adjustment depending on the manner in which the question was asked).

Additional information concerning concomitant instructional outcomes was derived from student answers to three questions. The questions asked students to state

what they "liked best," "disliked" or "found surprising" about their month-long classroom experience. These responses were summarized and analyzed descriptively.

QUESTION 3

Are there differences between the students under the cultural treatment and students under the regular treatment in the type and complexity of their spoken reaction to their learning in the trigonometry unit?

This question was answered by categorizing, quantifying and summarizing the taped responses of students made during the debriefing. The questions posed and other information relating to the quantification of students responses were previously discussed in this chapter. The analysis of the debriefing responses was descriptive.

Additional information concerning the efficacy of the cultural treatment was gained from the teachers involved in the study. The teacher questionnaires, as previously described, were summarized in order to assess the teachability of the cultural approach as perceived by them.

VI. LIMITATIONS OF THE STUDY

Several assumptions were made relative to this study. These assumptions include:

1. The presence of the researcher in the classroom affected student behavior in all classes in a similar

fashion.

2. All subjects interpreted the items on the achievement tests in the same way.

3. Subjects expressed their true feelings and attitudes on the questionnaires, the written responses and the oral interviews.

4. All student respondents were affected in the same manner by the presence of a tape recorder during the interviews.

The present chapter has indicated the nature of the sample, stated the research questions and the testable hypotheses, and outlined the analyses used. The following chapter reports the findings of the investigation.

CHAPTER V

RESULTS OF THE STUDY

The purpose of this study was to investigate the efficacy of a culturally based approach to a unit of trigonometry at the eleventh grade level. This chapter reports the data collected during the investigation, the results of testing the hypotheses and the other findings of the study. In presenting the results related to each question, the question or null hypothesis is stated, and the results of the analysis given.

QUESTION 1

Are there differences in achievement on the trigonometry unit between students under the cultural treatment and students under the regular treatment?

Hypothesis 1

> There is no significant difference between the mean achievement scores on the trigonometry unit between the regular and cultural classes within each school.

Results

This hypothesis was tested using analysis of covariance of the trigonometry achievement scores with the last report grade in Mathematics 23 as the covariate.

Table 2 compares the means of the last report grade (LRG) and the final achievement on the trigonometry unit (FAU) by classes. Table 3 summarizes the analysis of covariance performed on the appropriate scores.

TABLE 2

COMPARISON OF MEANS ON LRG (LAST REPORT GRADE) AND FAU (FINAL ACHIEVEMENT ON UNIT) BY CLASSES

				FAU	
School	Classes	n	LRG	Unadjusted	Adjusted*
A	AR	28	61.3	54.3	54.9
	AC	28	62.3	55.1	54.6
B	BR	12	58.0	58.6	60.7
	BC	26	61.7	61.0	60.0

*FAU mean adjusted because of covariate effect

TABLE 3

ANALYSIS OF COVARIANCE OF FAU SCORES BY CLASSES

School	Source	df	MS	Adj F	p
A	Group	1	0.86	0.006	0.94
	Within	53	138.13		
B	Group	1	1.38	0.03	0.87
	Within	35	51.27		

Hypothesis 1 was not rejected. No significant difference was found between the mean achievement scores on the trigonometry unit between classes AR and AC or

between classes BR and BC. As indicated by Table 2 the final achievement mean was considerably lower than the last report mean in classes AR and AC. There was little change between these means in classes BR and BC.

QUESTION 2

Are there differences in certain affective outcomes between students under the cultural treatment and students under the regular treatment?

Hypotheses 2(a), 2(b), 2(c) and 2(d)

> There are no significant differences between the questionnaire response scores of students in the cultural classes and students in the regular classes within each school, with respect to the
>
> (a) difficulty items (b) usefulness items
>
> (c) interest items (d) cultural relevance items.

Results

Chi-square tests were made on the frequency of "favorable" and "unfavorable" responses between the cultural and regular classes. Table 4 gives the percentages of "Unfavorable" (UNF), "Undecided (UND), and "Favorable" (FAV) responses made by the regular and cultural classes on each of the four category groupings, the value of chi-square (χ^2) testing the relationship between the treatment and the responses, and the probability range of the

TABLE 4

PERCENTAGES AND SIGNIFICANCE OF RESPONSES TO QUESTIONNAIRE CATEGORIES BY CLASSES

Item Category	Class	Percentage of Responses		
		UNF	UND	FAV
	AR (24)	37	13	50
	AC (21)	52	17	31
Difficulty		$.10 < p(\chi^2 = 2.30) < .20$		
	BR (9)	11	33	56
	BC (20)	60	7	33
		$.01 < p(\chi^2 = 6.63^*) < .02$		
	AR	13	30	57
	AC	16	23	61
Usefulness		$.99 < p(\chi^2 = .000) < .98$		
	BR	11	24	65
	BC	8	14	78
		$.50 < p(\chi^2 = .28) < .70$		
	AR	26	20	54
	AC	37	18	45
Interest		$.10 < p(\chi^2 = 1.89) < .20$		
	BR	22	19	59
	BC	25	21	54
		$.80 < p(\chi^2 = 0.03) < .90$		
	AR	13	29	58
	AC	17	23	60
Cultural Relevance		$.50 < p(\chi^2 = 0.30) < .70$		
	BR	18	38	44
	BC	15	27	58
		$.50 < p(\chi^2 = .31) < .70$		

* Significant at the .05 level

Note: Numbers in brackets indicate number of students per class completing questionnaires.

observed chi-square for schools A and B.

For classes AR and AC Hypotheses 2(a), 2(b), 2(c) and 2(d) were not rejected. For classes BR and BC Hypotheses 2(b), 2(c) and 2(d) were not rejected but Hypothesis 2(a) was rejected.

Difficulty Items: Inspection of the percentages in Table 4 indicates that students in the cultural classes in both school A and school B found the unit on trigonometry more "difficult" than did students in the regular classes and in school B this difference was significant.

Usefulness Items: A higher proportion of students in AC responded "favorably" to the usefulness items than did students in AR in 3 of the 4 test items, and a higher proportion of students in BC responded favorably to the usefulness items than did the students in BR in all 4 of the test items. In neither school however was the difference significant.

Interest Items: In school A students in AR responded more favorably to the interest items than did students in AC, whereas in school B no discernible relationship was found to exist between response to the interest items and group membership.

Cultural Relevance Items: In both schools the proportion of students making favorable responses to the

cultural items was higher in the cultural classes than in the regular classes but in neither school was the difference significant.

Analysis of Part II of the Questionnaire

Part II of the questionnaire required students to respond to questions concerning what they "liked best," "especially disliked" and "found surprising" about the unit of trigonometry studied. In addition they were asked to provide any further comments related to the instructional unit. Table 5 contains a summary of the answers given to each of the three questions (and the comments) by students in the various classes. The table indicates the major categories mentioned by students in response to the questions and the percentage of respondents within the classes who identified the particular categories. Occassionally the answer space was left blank.

Inspection of the percentages in Table 5 yields the following results:

Question 1: What did you like best about this unit on trigonometry?

One-quarter of the students in AR and AC and one-third and two-thirds of the students in BR and BC respectively stated that the applications of trigonometry were what they liked most about the unit. One-third of the students in the regular classes specifically stated that

TABLE 5

COMPARISON OF STUDENT-WRITTEN RESPONSES TO QUESTIONS CONCERNING THE INSTRUCTIONAL UNIT, BY CLASSES

Question and Number	Group	Major Categories Identified						
		Applications to real-world problems	Nothing	Understood Liked, Easy Interesting	Solving triangles	History	Other	Blank
1 What did you like best about this unit of work on trigonometry?	AR	25	38	25	13	0	17	3
	AC	24	14	38	10	5	14	16
	BR	33	33	33	0	0	11	0
	BC	65	0	10	0	0	20	15
		Nothing	Failure to understand parts of unit	Studied too much trig too fast	Solving problems and right triangles	History	Other	Blank
2 What did you especially dislike about this unit of work on trigonometry?	AR	29	25	17	0	0	25	0
	AC	29	24	19	5	0	10	10
	BR	22	0	0	22	0	44	0
	BC	45	5	5	20	10	15	5

Note: Entries indicate percentage of students identifying indicated category. Row sums may exceed 100.

TABLE 5 (Continued)

Question and Number	Group	Simplicity and power of trig to solve practical problems	Solving right triangles indirectly	Nothing	Realizing a measure of success	Historical problems and development of trig	Other	Blank
3 What did you learn while studying this unit on trigonometry that surprised you most?	AR	33	25	12	21	4	4	4
	AC	24	14	18	10	5	10	23
	BR	33	0	33	22	0	0	11
	BC	40	0	14	10	25	5	14
		Blank	Trig is important, useful math.	-Enjoyed -Easy -Interesting	-Trig not useful -Waste of time	Would have liked better if understood	Other	
4 Further comments	AR	29	17	21	21	8	4	
	AC	71	5	0	5	10	5	
	BR	55	22	0	21	0	11	
	BC	60	30	15	0	5	10	

Note: Entries indicate percentage of students identifying indicated category. Row sums may exceed 100.

there was "nothing" which they liked best about the unit whereas almost no-one in the cultural classes made similar statements. The historical dimension of the cultural classes was seldom identified as one of the characteristics of the unit best liked by the students.

Question 2: What did you especially dislike about this unit on trigonometry?

An equal proportion (29%) of students in groups AR and AC said that there was "nothing" which they particularly disliked whereas 22 and 45 percent of the students in BR and BC respectively made the same remark. As compared with students in school B, a relatively high proportion of both classes in school A especially disliked the speed with which the unit was covered and the resultant failure to understand all or parts of the content. Ten percent of the students in BC disliked the historical aspect of the unit.

Question 3: What did you learn during this unit on trigonometry that surprised you most?

At least one-quarter of all students mentioned the simplicity and power of trigonometry to solve real-world problems as the characteristic of the unit that surprised them most. The proportion of students who identified this characteristics was higher in AR (33%) than in AC (24%), whereas in school B the proportion of students identifying

this characteristic was higher in BC (40%) than in BR (33%). Consequently no trend was discernible across schools. Some students were most surprised by the degree of success they had achieved on the trigonometric unit. The proportion of students in the regular classes who expressed this view was double the proportion in the cultural classes who expressed a similar view. In school A, historical problems and related information were seldom mentioned. In school B however 25% of the students in class BC and 0% in class BR specifically mentioned historical problems or the historical dimension of the treatment as that which surprised them most.

4. Student Comments: High percentages of students in all groups left this space blank. Once again, students in all classes mentioned the importance and usefulness of trigonometry. As in Question 3, a higher porportion of students in AR mentioned this than did students in AC, whereas in school B a higher proportion of students in BC mentioned this fact than did students in BR. No one in groups AC or BR suggested the unit was enjoyable, easy, or interesting, whereas 21% and 15% of the students in AR and BC respectively stated this. Twenty-one percent of the students in the regular classes expressed the opinion that the unit was a waste of time and not useful or relevant to their interests. On the other hand, almost no one in the cultural classes suggested this. About 10% of

students in both classes at school A remarked that they would have liked the unit better had they understood it.

QUESTION 3

Are there differences between the students under the cultural treatment and students under the regular treatment in the type and complexity of their spoken reaction to their learning in the trigonometry unit?

The data used to answer this question came from an analysis of the tape-recorded answers of students to three undirected questions of the form "What have you learned by studying this unit on trigonometry?". A random sample of respondents was selected from each of the four classes to participate in the debriefing. The selection was made by drawing student numbers from a container. Table 6 indicates the percentage of students in each group sample interviewed who referred to the categories indicated.

The solution of right triangles using the methods of trigonometry was recalled by students more often than any other category, and in particular, the applications to real-world problems. No students in the regular classes recalled any historical items whereas about 15% of all students in the cultural classes recalled specific historical problems whose solution involved trigonometric methods. Except for the historical items there was no discernible

TABLE 6

COMPARISON OF RESPONSES TO DEBRIEFING AMONG GROUP SAMPLES FROM AR, AC, BR, AND BC

Areas Identified by Respondents		Percentage of Group Samples Identifying Areas AR (17)	AC (12)	BR (7)	BC (18)
Pythagorean theorem		6	8	14	6
Primary trig ratios		59	50	29	39
Trig tables		12	33	14	6
Solution of right triangles	F*	59	50	43	33
	A	71	50	86	83
	H	0	17	0	11
Field work		/	/	14	44
Graphs		/	/	0	17
Trig is practical useful, powerful		29	33	57	50
Trig was interesting, easy or "liked"		18	17	14	44
Learning nothing Trig is useless		12	25	0	0
History		0	17	0	22

Note: Numbers in brackets indicate sample size.

*F (Factual); A (Applications); H (Historical)

relationship between the type of material recalled by students and group membership.

In order to compare the complexity of the responses, the mean response weight (MRW) of each of the groups was calculated. They are displayed in Table 7. The MRW of AC was greater than the MRW of AR and similarly the MRW of BC was greater than that of BR. The MRW was also calculated for each group and classified according to the achievement grade in Mathematics 23 on the last report prior to the unit. The classification was arbitrarily set as above or below 60%. The resulting data are shown in Table 7.

TABLE 7

MEAN RESPONSE WEIGHTS (MRW) BY CLASSES AND ACCORDING TO GRADE ON LAST REPORT

		MRW According to Grade on Last Report	
Classes	MRW	$\leqslant 60$	> 60
AR	3.47	3.33	3.54
AC	3.58	2.75	4.00
BR	3.85	3.80	4.00
BC	4.55	4.40	4.75

In every class the MRW of students whose last report grade was less than or equal to 60%, recalled less information or less complex information than students whose last report grade was greater than 60%.

In response to the debriefing questions students often voluntarily expressed their feelings about and attitudes toward the trigonometric content of the course and/or the unit as a whole. Several students for example, suggested that they "liked" the unit, or that it was "interesting" or "easy." In groups AR and AC the percentage of students mentioning this was about 17, whereas in BR and BC the percentages were 14 and 44 respectively. In class BC, 28% of the students specifically referred to the unit as "interesting." In groups AR and AC respectively 12 and 25 percent of the sample interviewed stated that they had "learned nothing" or that the unit was of little or no value to them. Similar remarks were not made in either group BR or BC. About 20% of the students interviewed in the cultural classes mentioned the historical dimension of the presentation, whereas no students in the regular classes mentioned this.

Teacher Reaction Questionnaire

The purpose of this questionnaire was to determine the teacher's reaction to and evaluation of the instructional materials used, the cultural treatment as they perceived it, student reaction to the cultural treatment and other topics related to the instructional period. Following is a summary of the findings:

1. Concerning the appropriateness of the cultural treatment for Mathematics 23 students, one teacher thought

the treatment was "a bit heavy historically" but otherwise quite appropriate and meaningful, while the other teacher was less enthusiastic.

2. Both teachers observed that students responded positively to the cultural treatment. They stated that students particularly appreciated the trigonometric problems with a local flavor. One of the teachers remarked that "the history of trigonometry added a new and interesting dimension."

3. Teachers suggested that the cultural treatment could be improved by:

(a) providing more time for the teaching of the unit.

(b) adding more problems of local interest, or those indicating the applicability of trigonometry to "real-life problems."

(c) providing a problem set with greater variance from easy to difficult.

4. When asked if they would use any of the ideas and materials of the cultural treatment in future lessons on trigonometry, both teachers responded in the affirmative. One teacher planned to employ the ideas and materials "particularly" in Mathematics 30 classes, since, it was argued, "these people not only need this type of enrichment, but would probably appreciate it."

5. One teacher remarked concerning the cultural

approach: "It is a very refreshing direction to take in the teaching of mathematics."

CHAPTER SUMMARY

In this chapter two eleventh-grade trigonometry classes under the regular and cultural treatments in each of two schools were compared with respect to final achievement scores on the unit, the type and complexity of student response to their learning, affective outcomes of instruction and teacher reaction to the unit. Analysis of the data indicated the following conclusions and trends.

1. There was no significant difference in either school between the regular and cultural classes in achievement on the trigonometry unit. For both classes in school A the unit achievement mean decreased about 8% from the corresponding last report mean, whereas a comparison of these two means between the classes in school B showed little variance.

2. (a) In both schools students in the cultural classes found trigonometry more difficult than did students in the regular classes. In school B this difference was significant at the 0.05 level.

(b) In both schools a higher proportion of students in the cultural classes perceived trigonometry as being useful than did students in the regular classes, but in neither school was the difference between the

proportions significant.

(c) Students in the regular classes in school A responded more favorably to the questionnaire "interest" items than did students in the cultural classes, whereas in school B this pattern was reversed. In neither school was the difference significant.

(d) In both schools the proportion of students making favorable responses to the "cultural relevance" items was higher in the cultural classes than in the regular classes, but in neither school was the difference significant.

3. (a) At least 24% of students in each of the four classes stated that what they "liked best" about the trigonometry unit was learning that the topic found useful application in the solution of real-world problems. In school B, 33% of the students in BR stated this as compared to 65% in BC. In school A the differences were minimal. The historical facet of the cultural treatment was rarely identified as a "best liked" characteristic of the unit.

(b) In school A approximately the same proportion (.25) of students in both classes stated that their failure to understand parts of the unit as well as the speed with which the unit was studied was that which they especially disliked. These factors were rarely mentioned by either class in school B, but about 20% of the students in each class in school B suggested that they disliked the

problem-solving aspects of the unit. The historical dimension was disliked by 10% of students in BC.

(c) Students in all classes were surprised by the relative simplicity and power of trigonometric methods to solve practical problems. The proportion of students who identified this characteristic was higher in AR than in AC, whereas in school B this pattern was reversed. The proportion of students who were surprised at their perceived "success" on the unit was twice as high in the regular classes as in the cultural classes. Of the students in AC and BC, 5% and 25% respectively stated that the historical aspect of the unit surprised them most.

(d) Twenty-one percent of students in each of the regular classes expressed the view that the unit was a waste of time and not of benefit to them whereas almost no students from the cultural classes made similar remarks.

4. (a) Concerning the students spoken reaction to their learning (debriefing), only students in the cultural classes mentioned that they had learned about the history of mathematics or historical problems (about twenty percent of the AC and BC student sample interviewed made specific mention of this). Except for the historical items there was no discernible relationship between the type of material recalled and group membership.

(b) Of students interviewed, those in the

cultural classes had greater mean response weights than those in the regular classes and students whose last report grade was over 60% had greater mean response weights than students whose last report grade was less than or equal to 60%.

5. (a) In response to the debriefing interviews 17% of students in the AR and AC classes stated that they liked the unit, found it interesting, or easy, whereas in the BR and BC classes the percentages of students similarly were 14 and 44 respectively.

(b) In response to the debriefing, 20% of the students interviewed in the cultural classes mentioned the historical dimension of the instruction, whereas no students in the regular classes mentioned this.

6. (a) Both teachers involved in the study remarked that students responded positively to the cultural approach and particularly to the problems of local interest requiring trigonometric solutions.

(b) One teacher thought that the cultural approach was quite appropriate for Mathematics 23 students but suggested that the unit as designed may have placed too much emphasis on the historical dimension of the topic.

Analysis of the data reveals considerable variability in the results between the two schools. This will be discussed in the following chapter.

CHAPTER VI

CONCLUSIONS, DISCUSSION, AND IMPLICATIONS

Since the introduction of the "new mathematics" programs in the early sixties, courses at the secondary school level have been characterized by their emphasis on the logical structure of the discipline, deductive rigor, and the relatively early introduction of abstract mathematical systems. As a result of these innovations, criticisms were soon directed at some of the modern content as well as some of the very things which characterized the new programs. Critics believed that undue emphasis on the theoretical and structural nature of mathematics was inappropriate for most secondary school students and would leave them with a distorted and biased view of mathematics.

Some of the major critics proposed that an entirely fresh and more global view of mathematics be taken at the secondary level. This view would present mathematics as a cultural resource of man, invented by him and for him, in order to serve his ends and purposes, whether aesthetic or utilitarian. The approach advocated would present mathematics as a part of the greater body of all human knowledge, and would bring to the classroom not only the essential skills, techniques and processes of mathematics, but also its reason for being, its genesis, its creators,

its evolution, its interrelations with other areas of human endeavor, its usefulness, its central role in the historical development of societies and its crucial place in the modern world of the student.

Acceptance of the basic tenets of this "cultural approach" has been increasing steadily over the last twenty years. Recently, many of the characteristics and aims of the approach have been advocated by Coleman, Edwards and Beltzner (1975) as being desirable and important new directions for the teaching of mathematics in the secondary schools of Canada. Increased attention, they say, must be given to the attainment of the affective goals of mathematics instruction, long neglected as being of minimal importance.

Those who advocate the cultural approach to mathematics instruction suggest several benefits which should accrue as a result of such an instructional strategy. Most of these relate to the affective rather than the cognitive goals of mathematics instruction. Advocates suggest that if students learn mathematics within a meaningful context and come to recognize the central importance of the discipline in their lives and their culture, they will come to see mathematics as more meaningful, interesting and relevant to their existence. Some suggest, rather cautiously, that students may, as a consequence of improved attitudes toward the subject, be motivated to further

study, greater understanding or higher achievement.

I. THE STUDY

The purpose of this study was to investigate the efficacy of such an approach to mathematics instruction and to investigate the outcomes of instruction. This was done by designing a unit of mathematics at the secondary school level based on the basic tenets of the "cultural model" as outlined in the related literature, preparing appropriate instructional materials, teaching the unit in a school setting and evaluating the resultant outcomes of instruction.

Four Mathematics 23 classes, two from each of two Edmonton high schools participated in the study for a period of from three to four weeks. Trigonometry was selected as the area of study. In each school one teacher taught two classes of trigonometry, one from the cultural approach and the other from the regular approach. In each school the outcomes of instruction between the regular and cultural classes were compared relative to students' achievement and student and teacher reaction to the unit.

The analysis of the data obtained in this study resulted in the following conclusions.

II. CONCLUSIONS

1. No significant difference was found in the mean achievement scores on the unit between the regular and cultural classes.
2. (a) Students in the cultural classes in both schools found the trigonometry unit more difficult than did students in the regular classes. In school B the difference was significant.

   (b) No significant difference was found between the questionnaire response scores of students in the cultural classes and students in the regular classes in either school with respect to the questionnaire "usefulness" items. In each school however, the proportion of students who responded favorably to the usefulness items was greater for the cultural classes than the regular classes. In addition, data and responses obtained from other evaluative instruments used in this study indicated that in each school students in the cultural classes had a greater awareness of the usefulness of trigonometry than did students in the regular classes.

(c) No significant difference was found between the questionnaire response scores of stu- ents in the cultural classes and students in the regular classes in either school with respect to the "interest" items. Information and data obtained from other evaluative instruments however, indicated that students in class BC found their unit of trigonometry more "interesting" than did students in BR. In school A there was little discernible difference between the two group responses.

(d) No significant difference was found between the questionnaire response scores of students in the cultural classes and students in the regular classes in either school with respect to the "cultural relevance" items. In each school however, the proportion of students making favorable responses to the "cultural relevance" items was higher in the cultural classes than in the regular classes.

3. (a) The role, simplicity, and power of trigonometric methods to solve real-world problems was mentioned most often by all groups as the facet of the trigonometry unit which was "best liked" by students. In each

school the proportion of students stating this was higher in the cultural classes than in the regular classes. Few students in the cultural classes suggested that the historical dimension of the unit was what they "liked best." Ten percent of the students in the cultural classes stated that it was the historical facet of the unit which they especially disliked.

(b) Students in all classes were surprised and impressed by the power of trigonometric methods to solve problems of practical interest. The proportion of students mentioning this was higher in the BC class than in the BR class whereas this trend was reversed in school A. Relative to the historical dimension of the cultural treatment, one quarter of all students in the BC class mentioned this, whereas in the remaining three classes, it was seldom mentioned.

4. From the students' written comments it was noted that about one-quarter of the students in each of the regular classes expressed the view that the unit was a waste of time and not useful or relevant to their needs, whereas almost no-one in the cultural classes of either school made

this comment.

5. An analysis of the debriefing responses revealed the following:

   (a) Only students in the cultural classes recalled historical items. Otherwise there was no apparent relationship between the type of student response and group membership. A relatively high proportion of students in all class samples interviewed specifically referred to the applications of trigonometry.

   (b) In each of the two schools, students in the cultural class sample recalled more information or more complex information than did students in the regular class sample, and those whose last report grade was above 60% recalled more than those students whose last report grade was less than or equal to 60%.

   (c) An equal proportion of students in each of the classes in school A liked the trigonometry unit or found it interesting. In school B however, the proportion of students in the cultural class who found the unit interesting was three times as great as the proportion in the regular

class.

6. Both teachers involved in the study stated that student response to the cultural approach was positive. They suggested further that the approach could be improved by reducing the historical content of the treatment and adding carefully graded real-world applications of trigonometry.

   The opinions of the participating instructors relative to the appropriateness of this approach to mathematics instruction in general and to Mathematics 23 trigonometry in particular varied considerably.

Concluding Remarks

Students in the cultural classes indicated a more positive attitude toward the trigonometry unit than did students in the regular classes. Compared to the cultural classes, a relatively high proportion of students in the regular classes implied or specifically stated that they found the unit a waste of time, useless, or irrelevant to their needs. Analysis of the data also indicated that students in the cultural classes perceived trigonometry as being more useful and applicable to their needs and those of society than did students in the regular classes.

III. DISCUSSION

Differences in the outcomes of instruction between the two schools may be attributed to external time constraints and/or the effect of the teacher variable. The in-school instructional period agreed upon by both teachers involved in the study was four weeks. Because of various external constraints however, the instructional period in school A was reduced to three weeks. The prevailing student attitude seemed to be that they came out of one pressure situation and into another. Analysis of the student questionnaires for school A indicated that the students felt they were under considerable pressure to complete the unit at all cost. This was evidenced by their questionnaire response indicating that 25% of all students in school A disliked the speed at which the unit was studied, which in their opinion did not allow sufficient time for adequate learning. The fact that the mean achievement scores of both classes in school A dropped 8% from the last report grade mean seems to substantiate this concern. In this context then, the additional information and materials which made up the cultural treatment may have been seen by students in the cultural class as contributing to their frustration.

Another factor which may have had a significant

effect on the outcomes of the investigation was the teacher variable. The study was designed to allow for individual differences in classroom style between the participating teachers. The instructor's philosophical view of mathematics and the nature of mathematics, as well as his philosophy of mathematics education appeared to be a significant factor affecting the outcomes of instruction under the cultural treatment. Unless a teacher is sympathetic to the broader view of mathematics as well as the educational aims of those who advocate the cultural approach, he will no doubt find it difficult to pursue a course of action in the classroom which violates his own existing philosophical and psychological stances relating to mathematics and mathematics education. Concerning the significance of the teacher variable in curriculum reform, Griffiths and Howson (1974, p. 62) have stated: "no change in practice, no change in the curriculum, has any meaning unless the teacher understands it and accepts it."

In any event, the teacher and the students in school A reacted in a less favorable manner to the cultural treatment than did those in school B.

Student Achievement

No significant difference was found in the mean achievement scores between the cultural and regular

classes in either of the schools. Both classes in a given school studied the same mathematical content over an equal period of class-time. Since a considerable amount of time was taken up in the pursuit of non-content objectives, the fact that the mean achievement scores of the cultural classes were not significantly lower than the mean achievement scores of the regular classes can be considered as somewhat of a plus for the cultural treatment.

Affective Outcomes

(a) Difficulty Items:

Students in the cultural classes found the trigonometry unit more difficult than students in the regular classes and in school B the difference was significant. Since students in the cultural classes were not tested on the historical aspects of the treatment it seems unlikely that they found the overall approach "difficult." What seems more likely is that the "applications" dimension of the cultural approach was the cause of their comparatively unfavorable response to the questionnaire "difficulty" items.

The problems prepared for the cultural classes were of necessity more "wordy" than their counterparts in the regular classes. This required that students

extract the essential information and decide upon the appropriate mathematical model to solve the problem. The worksheets were designed so that the problems assigned the regular and cultural treatments were equivalent in the sense that they required the same mathematical methods and model for solution. The problems presented for solution to the cultural classes contained considerably more of what Skemp (1973, p. 29) referred to as "noise," and he points out that the greater the noise the greater is the difficulty of the mathematical task.

(b) Usefulness Items:

Even though the differences between the proportion of students favoring the usefulness items on the questionnaire were not significant between the regular and cultural classes in each school, a greater proportion of respondents in the cultural classes favored these items than did the regular classes. Other data substantiate this finding. Since the "usefulness" of trigonometry was learned through the problems solved by the students, a perusal of student response to this facet of their instruction gives an indication of their feelings regarding the usefulness of the topic. In school B the proportion of students in the cultural class who "liked best" the applications of trigonometry to real-world problems was double the proportion in the regular classes. Similarly, a greater

proportion of students in the cultural classes of school B were "surprised" at the simplicity and power of trigonometry than were those in the regular classes. A similar feeling was evidenced by student response in the "comment" section of the questionnaire. In school A the trends were less well defined.

(c) Interest Items:

Students in AR responded more favorably to the interest items than did students in AC, whereas in school B no discernible relationship was found between response to the interest items and group membership. In neither school was the difference significant. As a result of the debriefing however, 44% of the BC sample interviewed specifically stated that they "enjoyed," "liked," or found the unit "interesting." This compares with only 14% of the BR sample interviewed who made similar remarks. In school A an equal percentage (17) in each class referred to the unit in this manner.

(d) Cultural Relevance Items:

In both schools the proportion of students making favorable responses to the "cultural relevance items" was higher in the cultural classes than in the regular classes, but in neither school was the difference significant. A small proportion of students in BC remarked that they particularly disliked the historical aspect of the unit.

In both cultural classes students indicated that the historical dimension surprised them. Almost no students in the cultural classes suggested that the historical dimension of the unit was that which they liked best. It seems then that the overall reaction of the cultural classes to the historical aspects of the unit was one of surprise.

By way of summary, students in the cultural classes found the unit more difficult than did students in the regular classes. Positive trends in favor of the cultural treatment were apparent however, by comparing the reactions and responses of students in both the "usefulness" and the "cultural relevance" items. Response to the interest items indicated considerable variance between the two schools and classes.

Type and Complexity of Students' Spoken Reaction to their Learning

The debriefing interviews placed the responsibility on the student to assess what he had learned. It required students to state in their own words what they had learned and provided the researcher with an indication of the type of information which the student thought worthy of recall. Students in all classes made specific mention of the power and simplicity of trigonometric methods to solve seemingly difficult problems. Fifteen percent of the student sample interviewed in the cultural classes mentioned that they had

learned about the history of trigonometry whereas no one in the regular classes made this observation. Except for this category however, little discernible relationship was found between the type of response and group membership.

Students in the cultural classes recalled more information or more complex information than did students in the regular classes. According to Wittrock (1974, pp. 184-195), thematically organized structures of knowledge facilitate retention and especially if the structures are learner generated. Modern mathematics programs organize the learning around the hierarchical structure of the discipline. It is possible that an historical framework could also provide a structure upon which students could attach and relate their learning. Freudenthal (1973, pp. 74-76) has stated that "connected matter is faster learned and longer retained," but suggests that there are other connections besides the structural connections within the discipline. He believes that "reality is the framework to which mathematics attaches itself . . . and that for the non-mathematician, the relation with the lived-through reality of the learner" provides essential and meaningful connections between mathematics and the outside world. These connections, he claims, aid in understanding, meaning and retention of the mathematics learned. The cultural approach attempted to emphasize the outside connections of trigonometry with the real-world environment

of the learner.

Teacher Reaction to the Unit

Earlier in the chapter the significance of the teacher variable in this study was discussed. Only one additional item is discussed here. One teacher suggested that the cultural treatment was a "bit heavy historically." This may reflect an opinion based on the reaction of the students to the historical parts of the cultural treatment or it may indicate a degree of anxiety on the part of the instructor concerning his own knowledge and competence in the area of the history of mathematics. Cooney, Henderson and Davis (1975, p. 116) have observed that teachers of mathematics tend to ignore the historical dimension of the subject because they know mathematics better than they know the history of mathematics.

IV. IMPLICATIONS

Teachers

This study has shown that it is quite possible for a practicing secondary school teacher to design a unit of mathematics from a cultural perspective, to assemble and prepare appropriate classroom support materials and to successfully teach the unit in a classroom situation. Considerable time and effort is required to do the necessary research and assemble materials, but the rewards for both the teacher and the students are substantial.

This study has shown too that students can come to appreciate the power and simplicity of mathematics as a problem solving tool through considering problems that are meaningful to them or important to the efficient operation of society.

This study has indicated that the joint pursuit of the affective and cognitive goals of mathematics education does not jeopardize the achievement of the cognitive goals.

Teacher Education

In order for teachers to teach mathematics from the cultural approach in anything but a cursory fashion, their mathematical knowledge must be much broader than it presently is. This knowledge can be gained through the reading of appropriate books or by taking university courses in the history of mathematics. Coleman, Edwards and Beltzner (1975, pp. 102-103) have suggested that universities should offer "Cultural Mathematics" courses and give university credit for their completion. The goals of the courses would be to create interest in mathematics, shape basic attitudes, and provide for a broader view of the nature and role of mathematics in society. May (1971, pp. 99-103) has long insisted that mathematics teachers must know the history of mathematics, the origins of mathematical topics and its role in the scheme of things. In particular he states that it is not sufficient to know mathematics, the teacher must also know about

mathematics.

Further Research

The results of this study are not generalizable beyond the population of Mathematics 23 students of trigonometry. The results of this study however, suggest several related areas for possible research.

One of the teachers involved in this study suggested that the ideas used and materials developed for the cultural treatment might have been more appropriate for Mathematics 30 trigonometry classes. One obvious suggestion then, would be to teach the Mathematics 30 unit on trigonometry from a cultural approach, and to compare the resultant outcomes with a regular Mathematics 30 class in the same school.

Studies similar to this could be conducted using other mathematics topics such as probability, geometry, algebra, the conics, or calculus, to determine the feasibility of such an approach using these typical secondary school mathematics topics.

Freudenthal (1973, p. 77) believes that

> Children can learn all you want. It is another fact that they can forget it just as completely. A teaching experiment is irrelevant if it does not tell how deeply the taught material has settled, and how long it remains active. The depth of settling is nothing else than the connectedness of lived-through reality, and its persistence can be guaranteed by the strength of these reactions.

Since the cultural approach attempts to establish

relationships between mathematics and the real world, an interesting area of investigation would involve the study of the effect of the cultural treatment on retention.

The researcher is aware of the difficulty of adequately quantifying and measuring outcomes in the affective domain. In order to measure more precisely such outcomes it is suggested that the necessary and appropriate measuring instruments be well researched, designed and adequately tested.

It is also important that the teachers involved in the study have more than a cursory understanding of the aims, characteristics, intents and intended outcomes of the treatment. This can be assured only through adequate prior two-way communication between the researcher and the teacher.

Advantages and Disadvantages of the Design

Advantages. In a feasibility study such as this it is not necessary or desirable to strictly control all of the variables such as class selection, teacher instructional style, and test design and grading. The objective was to permit the participating teachers and students to proceed in as usual a fashion as possible so that any differences in the attainment of the cognitive and affective goals would result from the effect of the instructional content.

Disadvantages. As a statistical study, even though the data were analysed to reveal significant differences and/or trends, the lack of controls on several variables leads one to be cautious with respect to conclusions.

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APPENDICES

APPENDIX 1

TEACHER RESOURCE MATERIALS

Historical Summaries and an Outline of the Evolution of Trigonometry

Applications of Trigonometry

HISTORICAL ORIGINS AND EVOLUTION OF TRIGONOMETRY

1900 - 1600 BC - Mesopotamia

The ancient Babylonians had a number system based on 60 (sexagesimal measure), and used it extensively in their study of astronomy. We can attribute the eventual adoption of the degree, minute and second to this fact. It is interesting to note that an ancient Babylonian clay tablet, referred to as Plimpton 322 exhibits a table of squares of ratios of sides of right triangles. Modern scholars have shown that this tablet has essentially a table of $\sec^2 \theta$ for $45° \leq \theta \leq 31°$. We must not assume however, that the Babylonians were familiar with our secant concept, since they did not introduce an angular measure in the modern sense.

The Pythagorean Theorem whose discovery and proof is usually attributed to the Greek mathematician Pythagorus, plays an important role in trigonometry. Tablets from the Old Babylonian period show that the theorem was widely used.

1650 BC - Egypt

In 1858 a tourist purchased a 1' x 18' papyrus scroll from a Nile resort town. It turned out to be the most extensive mathematical document of ancient Egypt. It was copied by a scribenamed Ahmes who states that the material was derived from a prototype of about 2000 BC. It is possible that the knowledge may have been handed down from Imhotep, the almost legendary architect and physician to the Pharaoh Zoser, who supervised the building of his pyramid about 3000 BC.

Problem 56 of this document, called the Rhind Papyrus, is of special interest in that it contains the rudiments of trigonometry and a theory of similar triangles. In the construction of the pyramids it had been essential to maintain a uniform slope for the faces and it may have been this concern that led the Egyptians to introduce a concept equivalent to the cotangent of an angle, ie. the reciprocal of the ratio of the 'rise' to the 'run'. The knowledge in extant Egyptian papyri is mostly practical in nature, and calculation was the chief element in the questions. It appears that Egyptian mathematics may have stagnated for a period of some 2000 years after a rather auspicious beginning.

ARENA OF GREEK MATHEMATICS

Places linked with mathematical luminaries of ancient Greece are going from west to east. Syracuse (Archimedes), Elea (Aeno), Crotona (Pythagoras) Miletus (Thales) and Alexandria (Euclid, Apollonius Hypatia, Eratosthenes, Hipparchus, Claudius Ptolemy, Heron)

300 BC - 200 AD GREEK TRIGONOMETRY

When the centre of Greek civilization moved to Alexandria, Greeks came in close contact with the Egyptians and the Babylonians, and hence the wealth of astronomical observations made by them became more accessible. During this 'Alexandrian period' and under the reign of the Ptolemys, great encouragement was given to intellectual endeavour. Among these were the constructing of a great home for scholars, called the Museum, and the equipping of a famous library. Such great men as Euclid, Archimedes, Claudius Ptolemy, Aristarchus Appolonius, Hipparchus, Diophantus, Heron and Eratosthenes all laboured out of Alexandria! For an excellent reference on ancient Alexandria, see "City of the Stargazers" by Kenneth Heuer.

Astronomy became a science at Alexandria, and under the studies of primarily two men: Hipparchus (150 BC) and Claudius Ptolemy (150 AD). The mathematical basis that Hipparchus and Ptolemy created for their work is today known as trigonometry.

Hipparchus was probably the most eminent astronomer of antiquity. He introduced into Greece the division of a circle into 360°, and is known to have advocated location of positions on the earth by latitude and longitude. Hipparchus has been credited with a 12 book treatise dealing with the construction of a 'table of chords'. These tables are essentially a table of

sines, the first trigonometric tables. Evidence shows that Hipparchus, called the "Father of Trigonometry" made systematic use of his tables to find the straight line distances across the heavens. Though his writings have been lost, he is reported to have catalogued 850 fixed stars.

Claudius Ptolemy (150 AD) wrote several works which were encyclopedic in nature. His thirteen volume work on astronomy called the "Almagest" (the greatest) by his successors remained the standard of excellence in astronomy until the time of Copernicus 1000 years later! Even though much of his work was similar to that of Hipparchus, most of his work remains today. Ptolemy gives us a 'table of chords' which essentially yields the sines of angles between 0° and 90° by 15' intervals. He was able to catalog 1022 stars. Ptolemy was also a great geographer. He wrote eight books that deal primarily with mapping the known world by latitudes and longitudes (a first). Ptolemy stated:

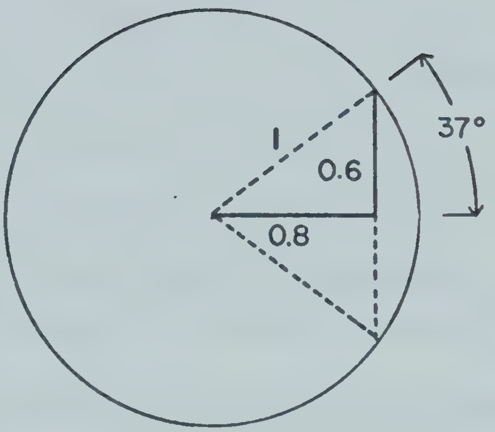
"I know that I am mortal and ephemeral, but when I scan the multitudinous circling spirals of the stars, no longer do I touch earth with my feet, but sit with Zeus himself, and take my fill of the ambrosial food of the gods".

Heron of Alexandria (100 AD) is probably best known for the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$, where A is the area of a triangle whose sides a,b,c, are known. It is apparent that in order to utilize this formula, the knowledge of an algorithm to evaluate the square root is required. Heron proposed an iterative procedure which has since been credited to Newton. Heron was a clever mathematician-engineer and did much to furnish a scientific foundation for engineering and land surveying. He invented and described applications of an ancient form of surveyors transit (theodolite). Heron startled his colleagues with a display of the power of trigonometry when he showed them how to dig a tunnel under a mountain by starting at both ends at once and having the two borings meet each other.

500 AD; INDIA

The birth of the sine function is credited to Hindu astronomers by

their use of the half chord. The work of Aryabhata (born in 476 AD) contain tables relating to half a chord(or half an arc) and half of its central angle. They had in effect calculated the coordinates of points on the unit circle corresponding to each degree of the circle. It appears that the fundamental ideas of the modern circular function is contained in these works. The ingenious methods used by the Hindus to calculate the half chords are related in Sawyers' book, "A Search for Pattern" (1970).



800 - 1000AD; ARABIA

Abu'l Wefa (940-998) provided a table of tangents and used all six of the common trigonometric functions. These results were derived from the unit circle and recorded in sexagesimal measure, even though the Arabs worked their arithmetic in decimal notation. By the time the Arabic documents reached Europe, the subject matter of trigonometry was better organized with proofs given for the half-angle, double angle, and other theorems Gradually decimals began to replace sexagesimals in the trig tables.

1400 - 1500 AD; EUROPE

It is apparent that astronomy has long contributed to the development of mathematics, in fact at one time the name 'mathematics' meant an astronomer. As the focus of activity in astronomy moved to Europe so also did the evolution of trigonometry. The computation of tables and the discovery of functional relations between parts of triangles was continued in the West.

Rheticus (1514-1576) spent twelve years with hired computers compiling 10 place trigonometry tables of all six trig functions. He was the first to define the trig functions using ratios of the sides of right triangles instead of the arc length or chord length of circles.

Francois Vieta (1540-1603) was the greatest French mathematician of the 16th century. He developed methods for solving plane and spherical triangles with the aid of all six trig functions. He obtained expressions for cos $n\theta$ as a function of cos θ. In addition Vieta observed that a trig ratio could be used to solve an algebraic equation, i.e., a series of numbers in a trig table could represent successive values taken on by a variable. This led to the trigonometric equation $y = \sin x$ and the modern functional idea. The transition to analytic trigonometry then was stimulated by Vieta who applied algebra and symbolism to trigonometry. He did much to convert trigonometry into a branch of pure mathematics. Trigonometric identities of various sorts were appearing about this time in all parts of Eurpoe, resulting in reduced emphasis on computation in the solution of triangles and more on analytical functional relationships. It was during this time that the name trigonometry came to be attached to the subject.

Descartes (1596-1650). The scope of trigonometry became even broader when Descartes invented the new graphing techniques based on a rectangular coordinate system. Now each curve possessed its own equation and vice versa. The association of equation and curve was a new concept that revolutionized mathematics. Thus, an equation like $y = \sin x$, could now be accurately graphed point by point to create a unique curve. By 1635, the first graph of half an arch of the sine curve had been drawn. Soon after, trigonometric functions were defined for real values of the independent variable.

Through the works of such men as John Wallis (1616-1703) Isaac Newton (1642-1727), Johann Bernoulli (1667-1748), and Leonhard Euler (1707-1783), trigonometry became involved in new theoretical developments due to the invention of the calculus. By the end of the 18th century Euler and others had exhibited all the theorems of trigonometry as corrollaries of complex function theory.

HISTORICAL SUMMARY

Trigonometry grew out of the practical needs and interests of Greek astronomers, where a conception of planets moving in circular orbits developed a need for determining the lengths of chords of circles if one knew the length of the corresponding arcs. Hindu mathematicians of about 500 AD saw greater utility in tables of half-chords and not only computed them but gave us the word "sine". As time went on more abstract and general definitions were given

to the trig functions, definitions that did not depend on the notions of angles, triangles or even necessarily on circles or arcs. It is a significant fact of the evolution of mathematics in general and trigonometry in particular that the more abstract and general definitions are the ones that particularly stress the special properties of the trigonometric functions, among which is the particularly important property of periodicity.

UBIQUITOUS TRIGONOMETRY

Trigonometry has gained for itself a permanent position in the world of mathematics because of two seemingly unrelated components or characteristics.

a. its power as a tool for spacial mensuration, and
b. its power as a precise mathematical model for situations which are periodic.

A. Measurement

1. Astronomy:
   Trigonometric methods are used to calculate:
   - the diameter of the sun, moon and earth
   - the distance of the earth from other planets
   - the distance between stars and planets
   - the distance between cities on the earth
2. Navigation:
   The techniques of mensurational trigonometry find application in:
   - plotting and maintaining flight plans of aircraft
   - charting and maintaining the courses of ships
3. Surveying:
   The tools of trigonometry enable man to:
   - define very accurately the property lines of lots, farms, towns, cities, provinces and nations.
   - lay out streets, sewers and street lights for communities
   - construct highways, railroads, bridges and tunnels
   - calculate the height, width or depth of inaccessible objects
   - calculate using indirect methods, areas and volumes

4. Engineering and Technology:

   Trigonometry finds application in engineering and technology in a thousand different ways. A few are listed below:

   - to evaluate the strength of building materials
   - to evaluate the stresses and strains on all types of structural components
   - to calculate forces, friction and torque.

5. Other:

   - carpenters, electricians, tinsmiths and stone masons use varying amounts of elementary trigonometry.

In light of the various applications of trigonometry as a science of measurement, it appears that Plato was correct when he stated: "The world can be made intelligible in terms of right triangles."

B. Periodicity

A.N. Whitehead (1964) observed:
"the whole life of Nature is dominated by the existence of periodic events, that is, by the existence of successive events so analogous to each other ... that they may be termed recurrences of the same event." The following examples of oscillatory (periodic) quantities from the physical, biological and social sciences as well as economics find their precise mathematical model in trigonometry.

1. Physical Science:

   - vibrating strings (any stringed instrument)
   - vibrating membranes (drums)
   - vibrating air columns (organs)
   - vibrating springs (clocks and watches
   - sonar, radar, microwaves
   - sound, light and water waves
   - heat transfer (thermodynamics)
   - alternating electric current (A.C.)
   - oscillating pendulums and bobs
   - motion of a piston within a cylinder

The following natural phenomena result from the periodic nature of our solar system:

- tides
- phases of the moon
- successive days, seasons and years
- predictability of the orbits and relative positions of the earth, moon, sun, planets and stars.

2. Biological Science:

Our bodily life is essentially periodic. It is dominated by the beatings of the heart, and the recurrence of breathing. Even plant life obeys recursive laws, laws which are determined primarily by the periodic nature of the "night-day" and seasonal occurrences.

3. Economics:

Economic activity seems to run in cycles. Economists refer to periods of expansion, retrenchment, recession and recovery.

In light of these many examples the following remark by Whitehead (1964) seems appropriate:

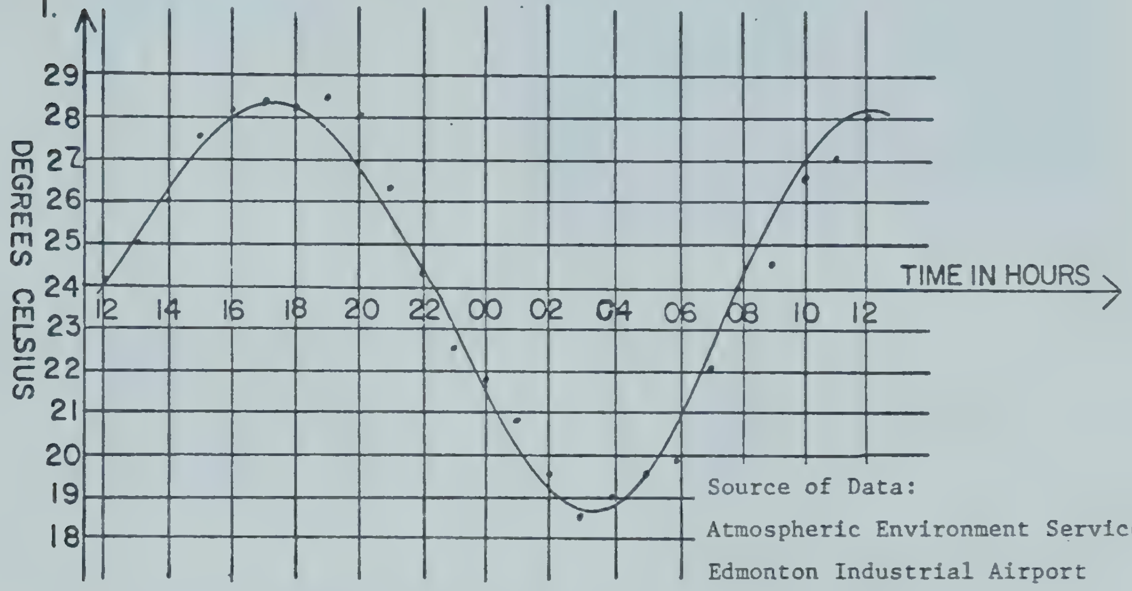
"It is evident that one of the first steps to make mathematics a fit instrument for the investigation of Nature is that it should be able to express the essential periodicity of things."

Trigonometry provides such an instrument!

C. Graphs that Illustrate the Periodic Nature of Certain Natural Phenomena

Hourly temperatures in city of Edmonton from noon July 2, 1975 to noon July 3, 1975.

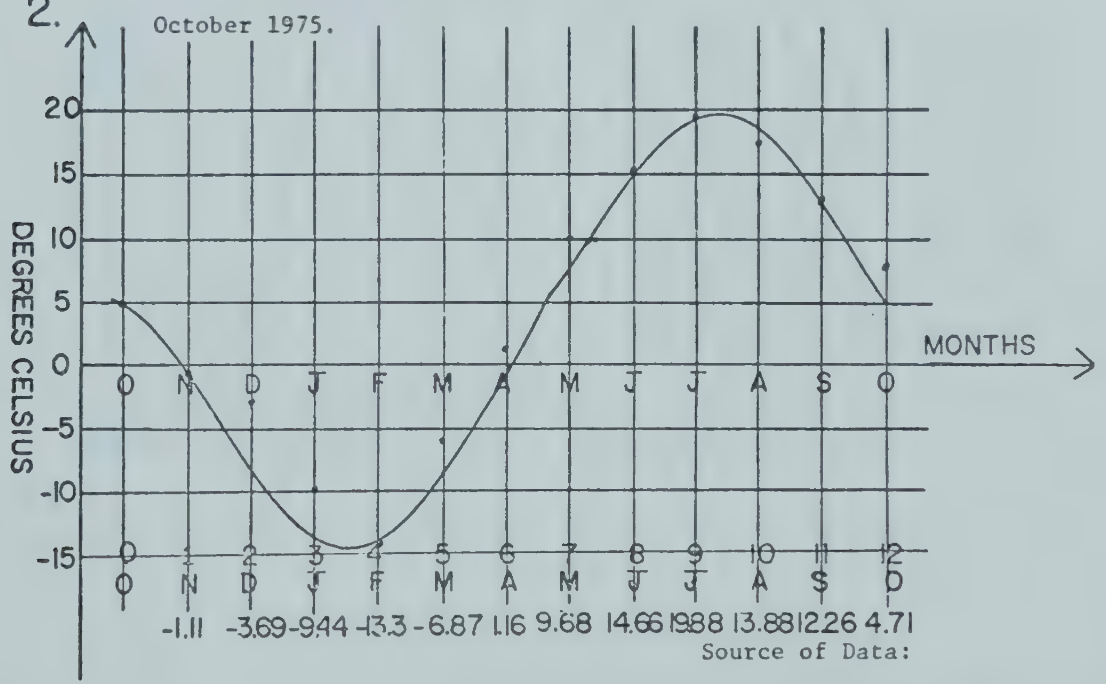
1.



Source of Data:
Atmospheric Environment Service
Edmonton Industrial Airport

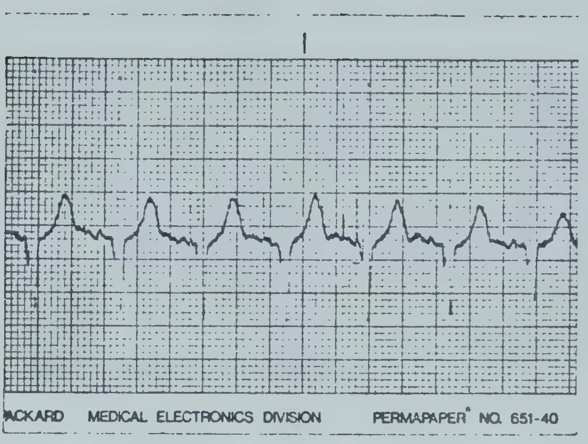
Average Temperature in city of Edmonton October 1974 to October 1975.

2.

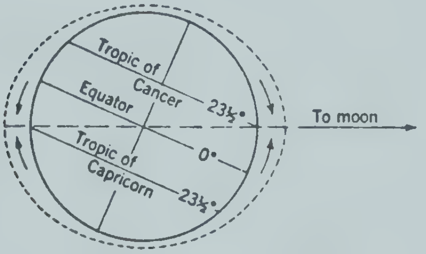


Source of Data:
Atmospheric Environment Service
International Airport
Edmonton, Alberta

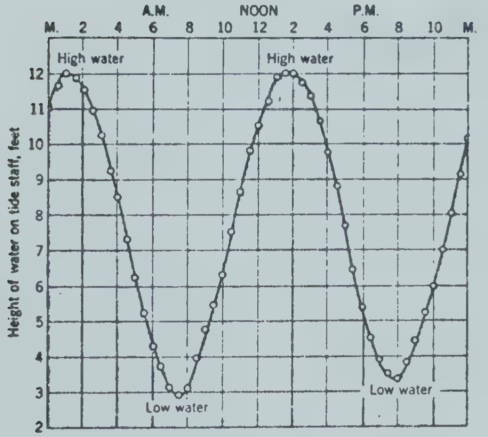
3. Graph of the Normal Heartbeat of a thirteen year old boy



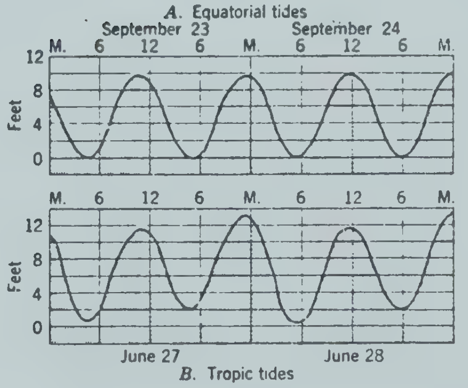
4. Height of water at Boston Harbor for a twenty four hour period



The moon's changing declination influences

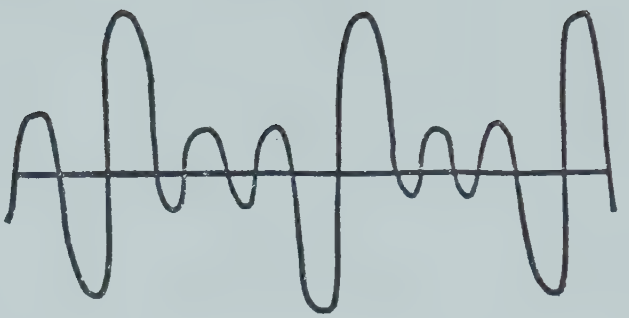


This graph shows the height of water at Boston Harbor measured every half hour for a 24-hour period, April 1, 1922. (After H. A. Marmer.)

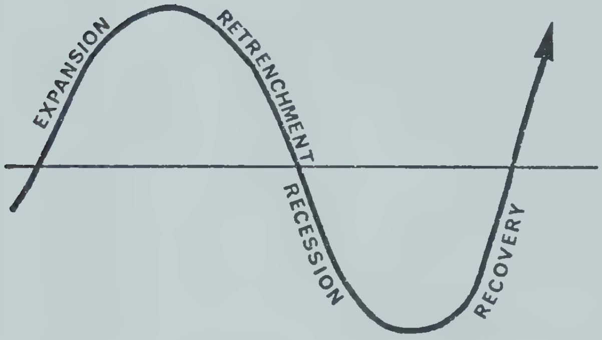


Changes in the moon's declination are reflected in these tide curves. (After Rude.)

5. Regular Pattern of the sound of a trumpet produced on a oscilloscope screen



6. The Business Cycle



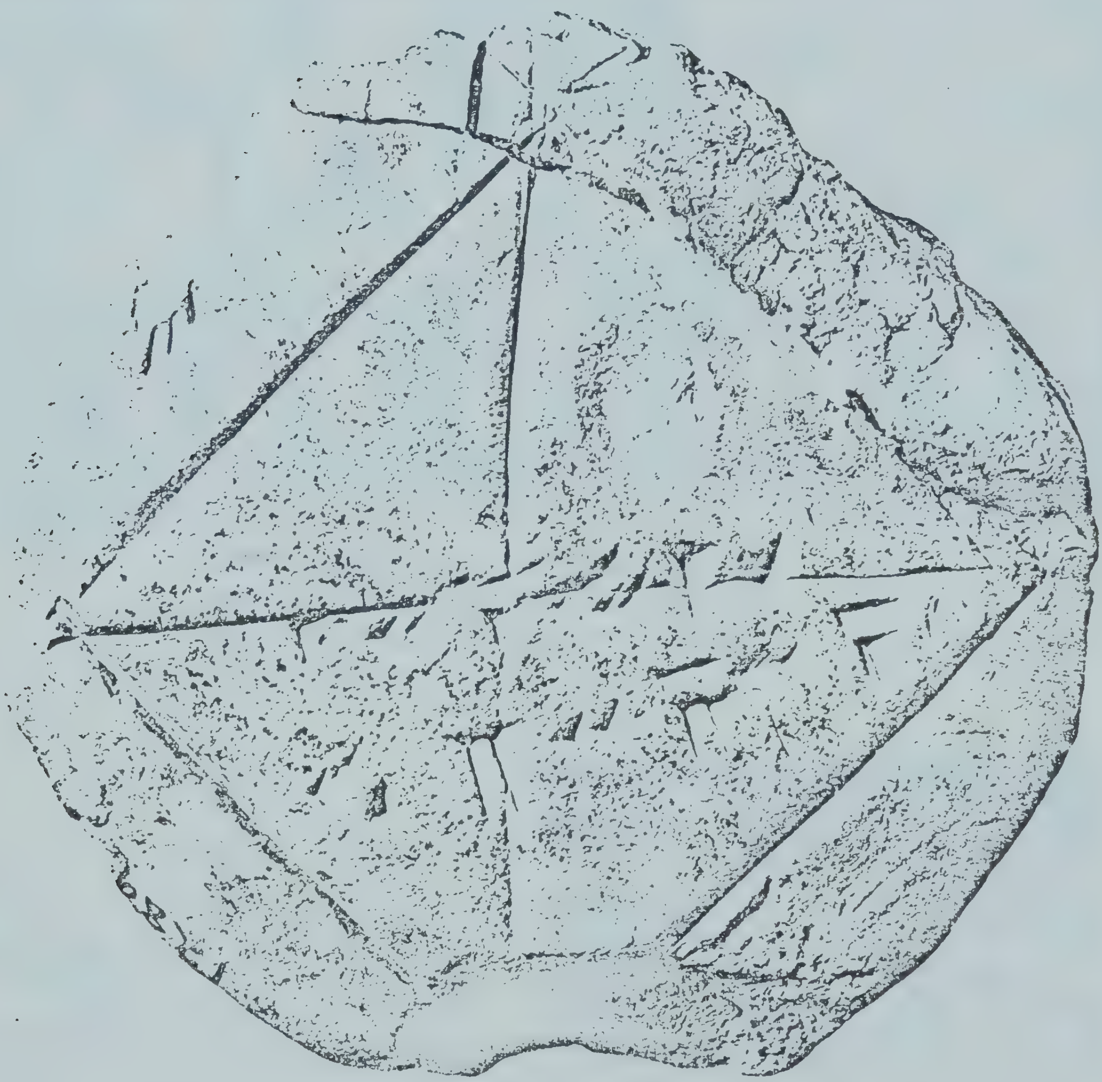
APPENDIX 2

PROJECTUAL MASTERS

THE ARENA OF ANCIENT BABYLONIAN, EGYPTIAN AND GREEK MATHEMATICS

T1

BABYLONIAN CLAY TABLET INDICATING A KNOWLEDGE OF THE PYTHAGOREAN THEOREM (1000 B. C. ?)

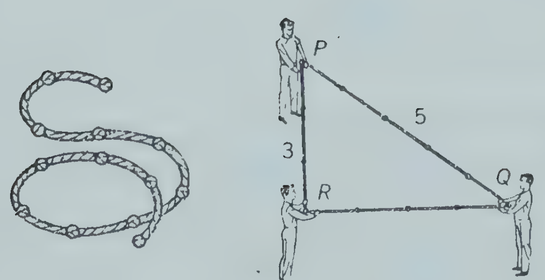


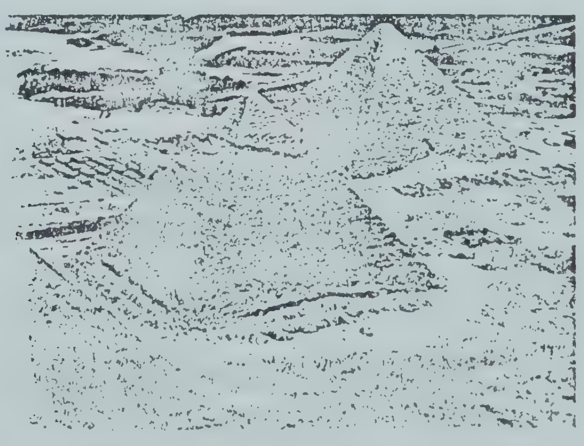
IN THIS TABLET A SQUARE AND ITS DIAGONALS ARE REPRESENTED. THE SIDE IS GIVEN AS 30, AND THE LENGTH OF THE DIAGONAL AS 42.2535.

T2

PRACTICAL EGYPTIAN MATHEMATICS
FOR FARMERS, SURVEYORS AND PYRAMID BUILDERS
1000 B.C.

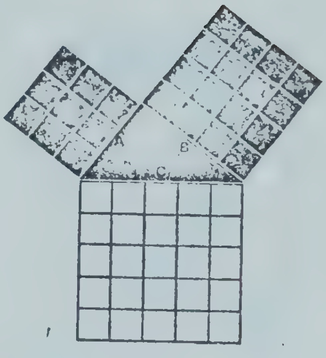
PART OF 3000 YEAR-OLD EGYPTIAN MURAL, INDICATES USE OF A ROPE DIVIDED INTO 12 EQUAL LENGTHS IN ORDER TO FORM A RIGHT ANGLE.





T3

PYTHAGORAS AND THE PYTHAGOREANS
(550 B. C.)

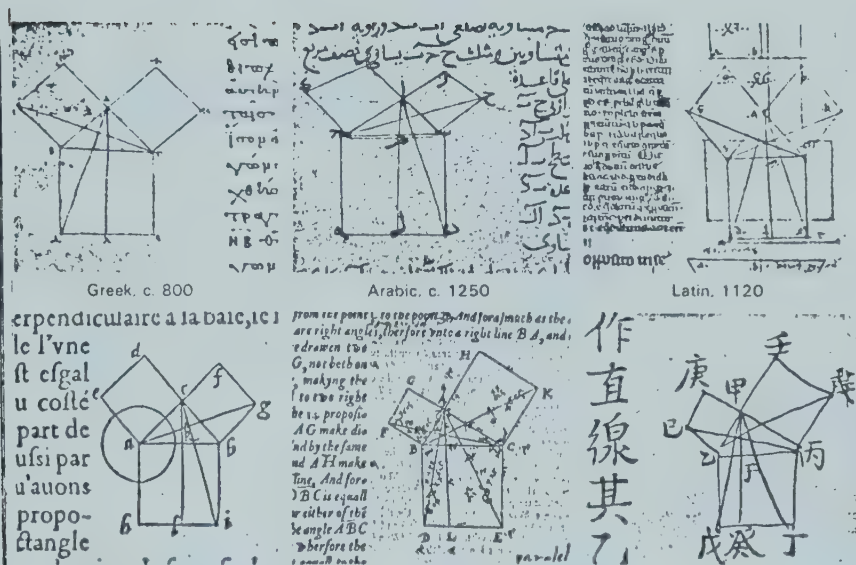


THE PENTAGRAM

- outstanding Greek mathematician and mystic
- travelled in Egypt, India and Babylon where he no doubt learned much mathematics
- founded a school and brotherhood in southern Italy know as "the Order of Pythagoreans"
- a semireligious cult developed which became very secretive and mystical
- Pythagoras is given credit for the proof of the "Pythagorean Theorem"

T4

PYTHAGORAS AND HIS THEOREM



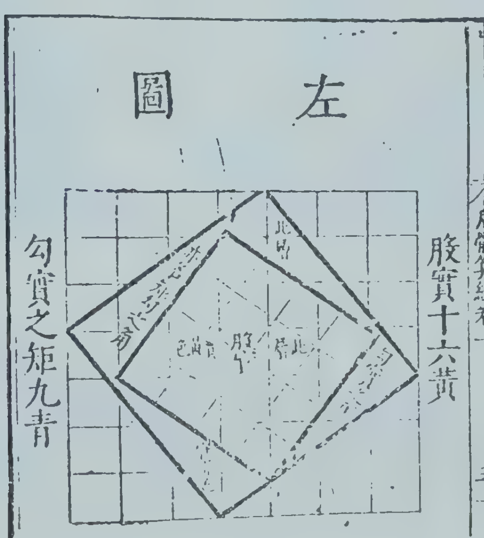
PROOFS OF THE THEOREM FROM SIX DIFFERENT CULTURES

A TEST OF BELLS AND WATER

A TEST OF STRING TENSIONS

A TEST OF COLUMNS OF AIR

Pythagoras discovered the underlying mathematics of the musical scale.

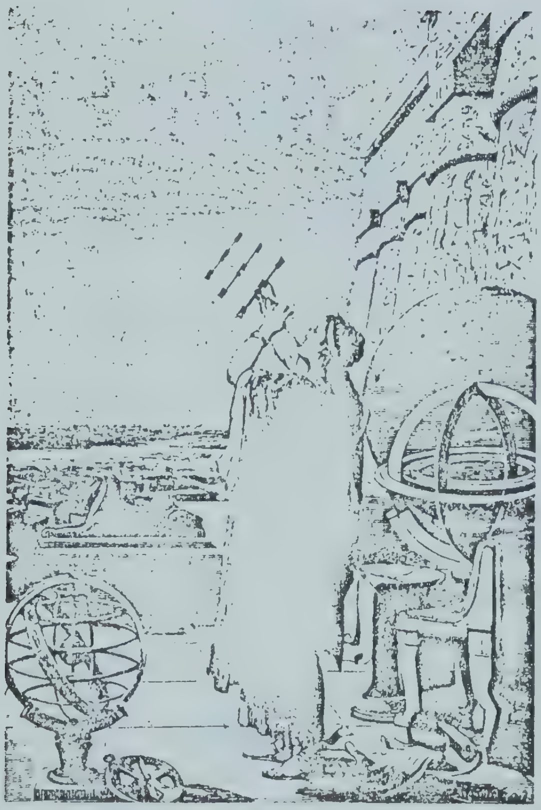


CHOU PEI
Contemporary of Pythagoras

T5

HIPPARCHUS - 150 B.C.

Early astronomer, Hipparchus, in observatory at Alexandria.



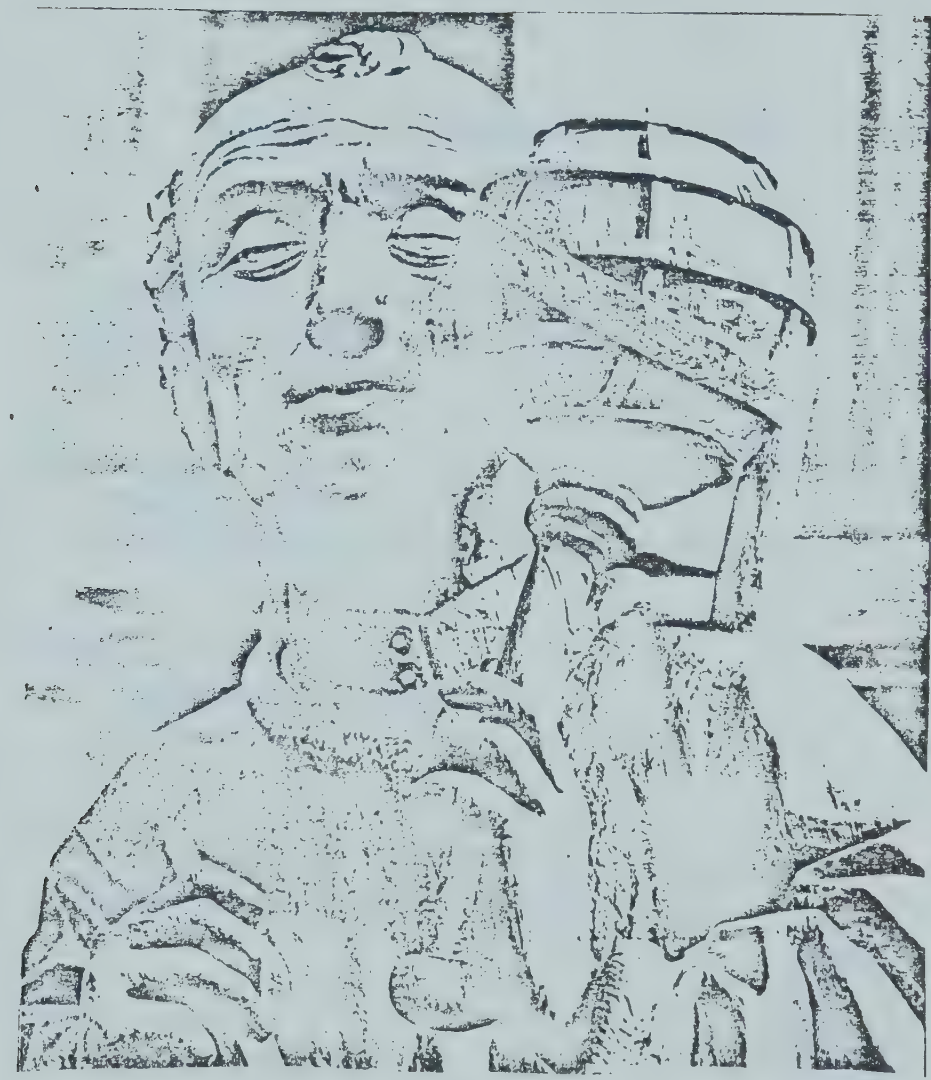
- outstanding Greek astronomer
- worked in Alexandria, northern Egypt
- called the "Father of Trigonometry"
- wrote several books on astronomy
- none exist today
- catalogued over 800 stars

Note:

Trigonometry began as a response to the practical needs of astronomers, the need to measure the universe and the distance between the stars. At one time "mathematics" meant "astronomer".

T6

CLAUDIUS PTOLEMY - 150 AD



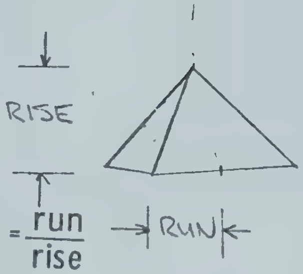
- great Greek astronomer from Alexandria
- built upon the work of Hipparchus
- wrote several large treatises on astronomy, most of which exist today
- his works were unparalleled for 1000 years (until Copernicus)
- catalogued over 1000 stars

T7

EVOLUTION OF TRIGONOMETRIC TABLES AND METHODS OF CALCULATION

EGYPT - 2000 B.C. :

- used a ratio of "run:rise" (the reciprocal of tangent ratio) in reference to the "slope" of the sides of the pyramids.



"seqt" = $\frac{\text{run}}{\text{rise}}$

HIPPARCHUS - 150 B.C.

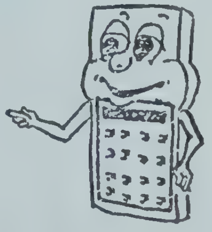
- Greek astronomer of Alexandria
- devised the first known table of <u>sines</u> - record lost

PTOLEMY - 150 A.D.

- Greek astronomer of Alexandria
- first recorded table of sines

RHETICUS - 1550 A.D.

- A European: spent 12 years with hired (human) calculators compiling 10 place trigonometric tables.



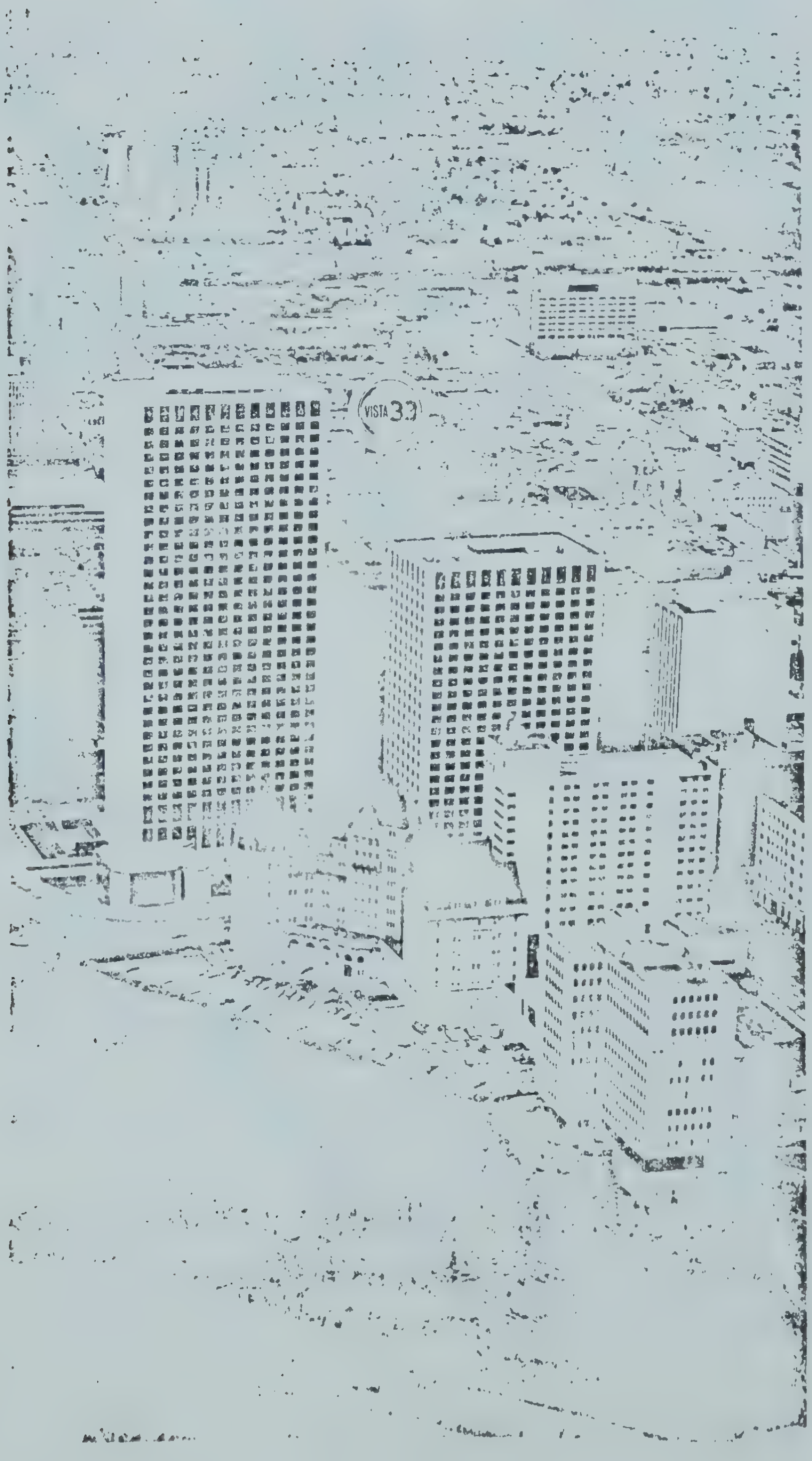
TODAY

- slide rules
- pocket calculators
- computers

Note - evolution of calculating devices from hand to mechanical to electronic. - compare calculation time and accuracy.

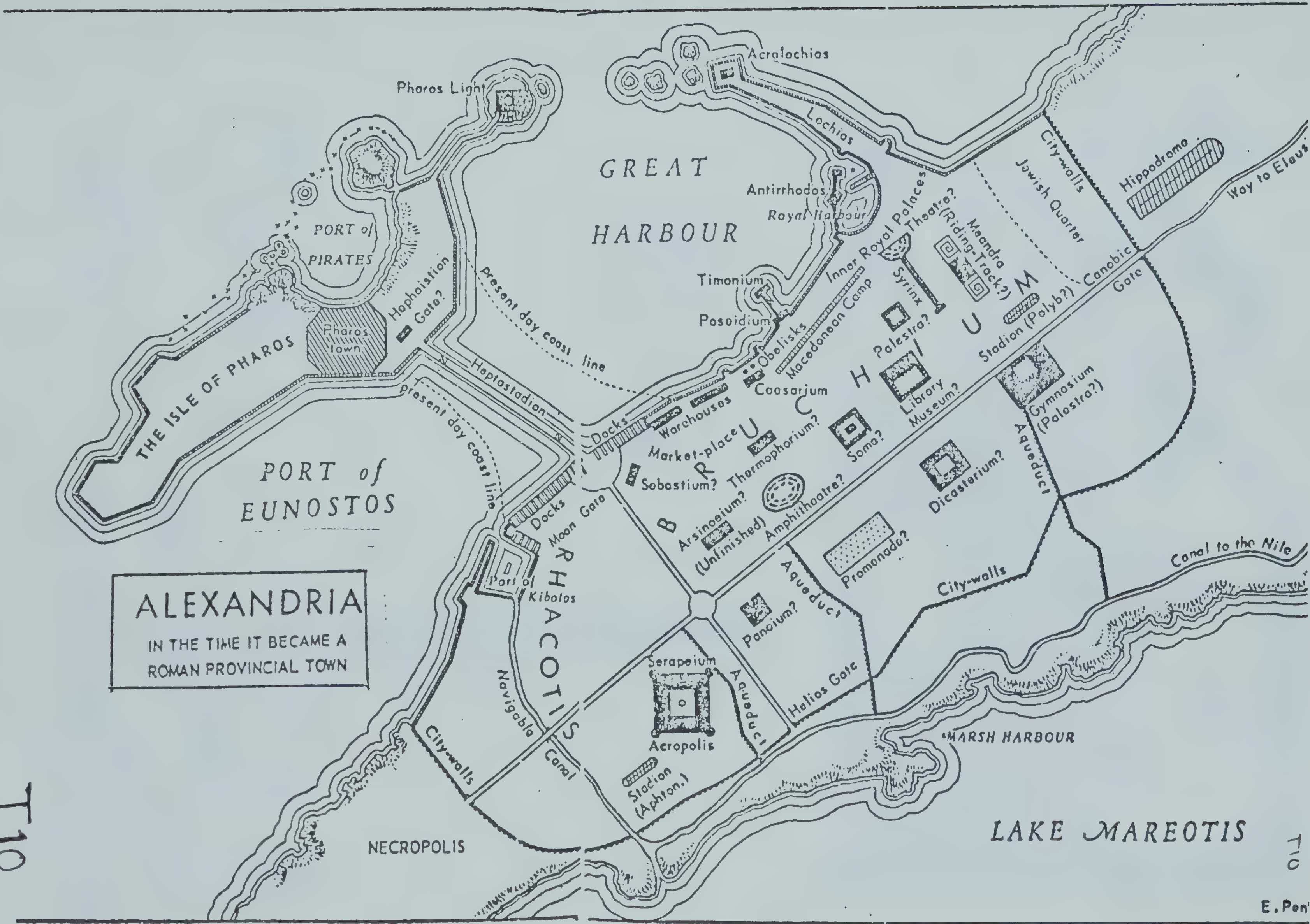
T8

HOW HIGH IS THE
ALBERTA TELEPHONE TOWER?



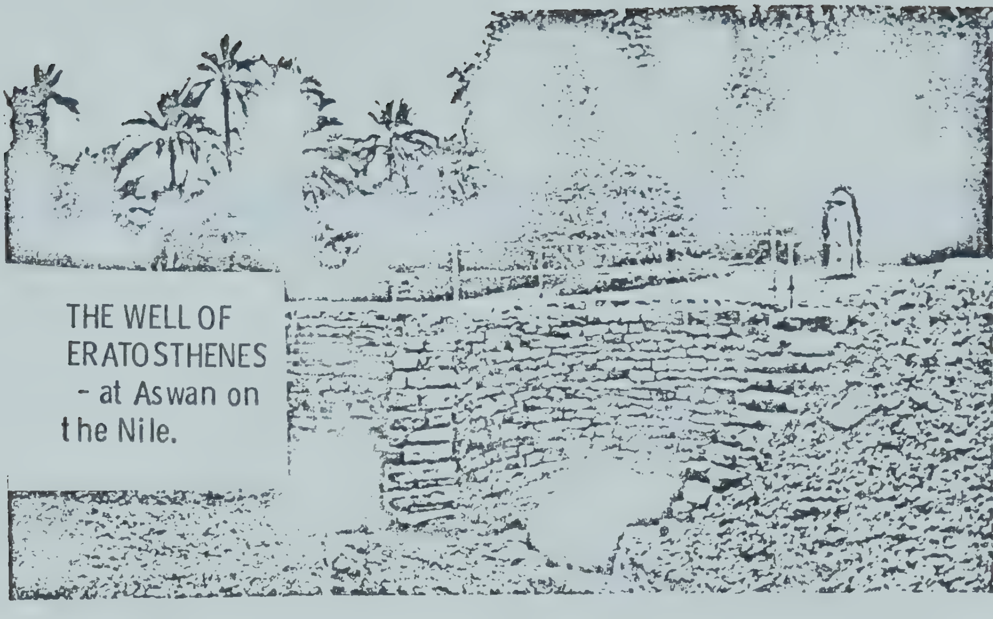
T9

T10



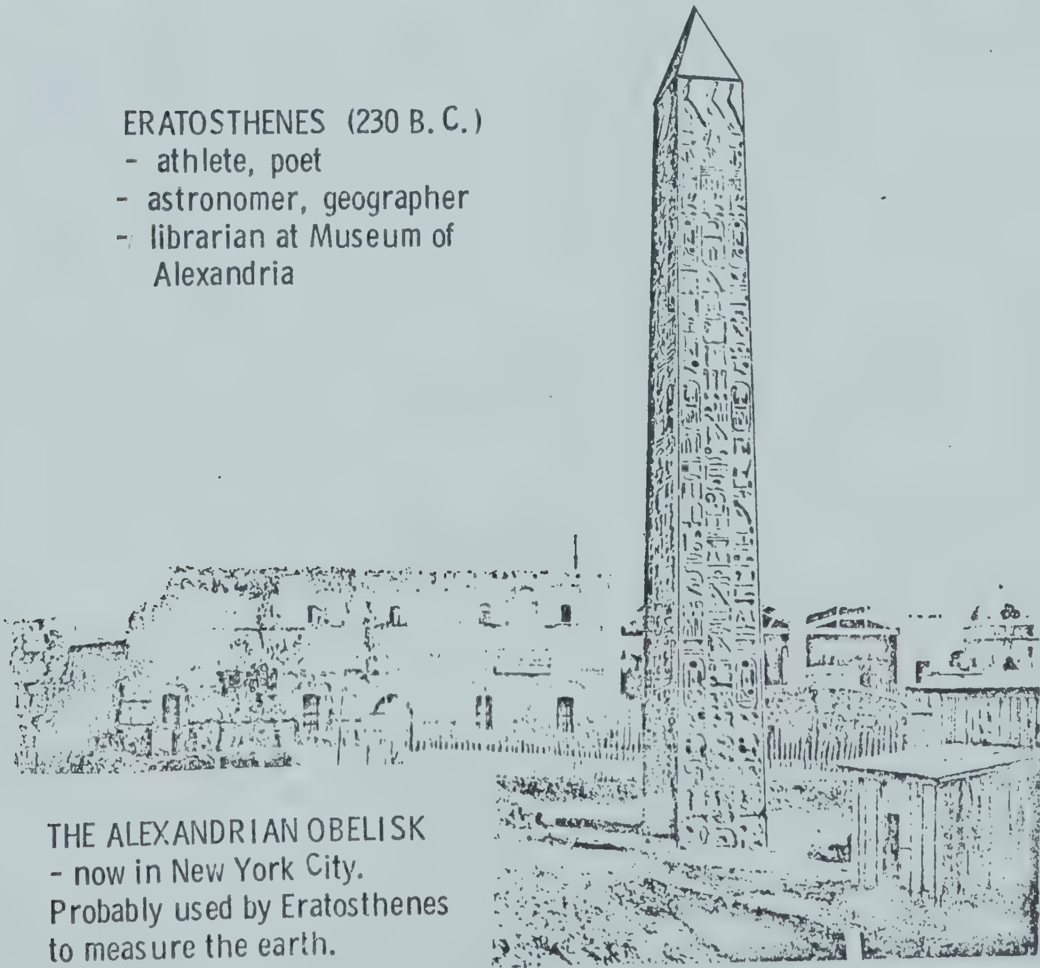
Alexandria. Delineating the city in the time it became a Roman

for the spectators, which was famous throughout the East. The Jewish quarter is



THE WELL OF ERATOSTHENES
- at Aswan on the Nile.

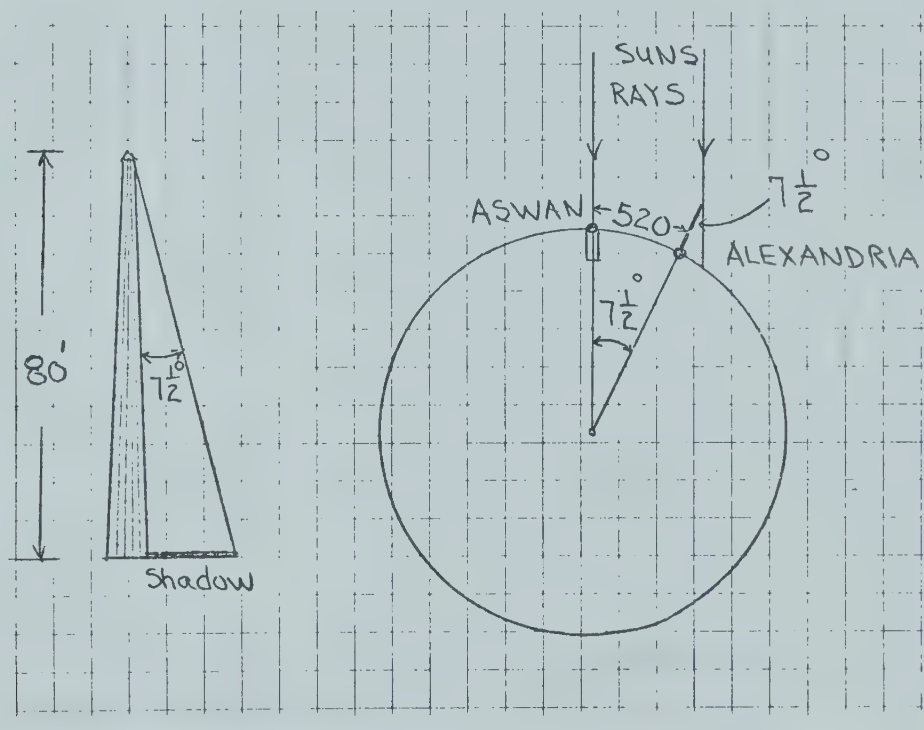
ERATOSTHENES (230 B. C.)
- athlete, poet
- astronomer, geographer
- librarian at Museum of Alexandria



THE ALEXANDRIAN OBELISK
- now in New York City.
Probably used by Eratosthenes to measure the earth.

T11

HOW ERATOSTHENES MEASURED THE EARTH USING A WELL AND A STATUE
(230 B.C.)

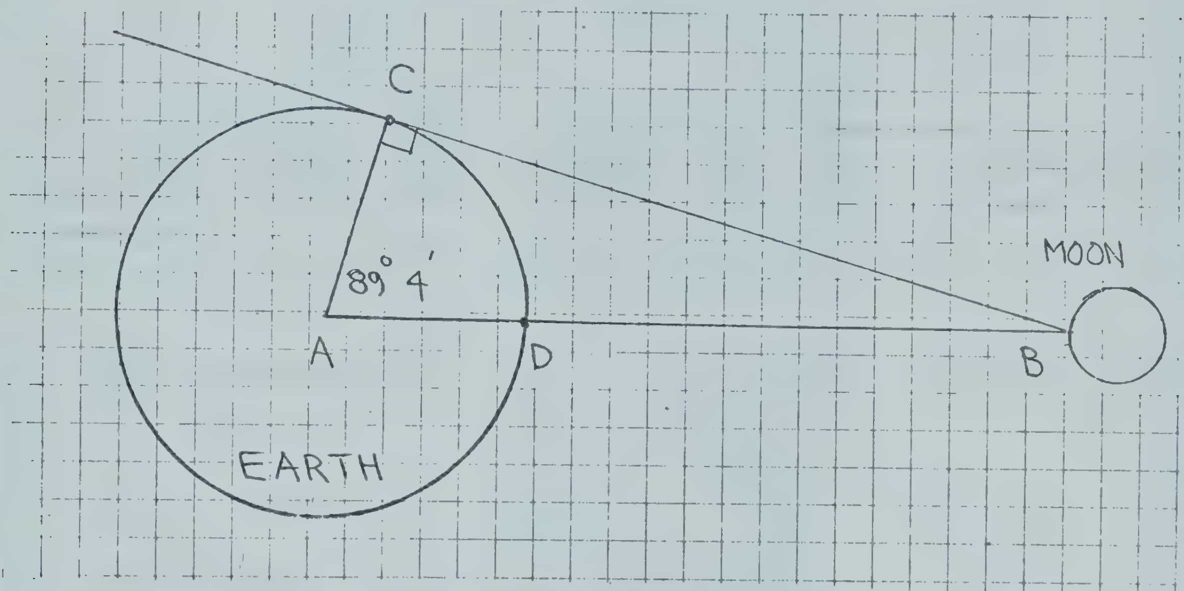


$$\frac{520}{7.5^0} = \frac{C}{360^0}$$

$$\therefore C = \frac{520\,(360)}{7\,1/2} = 24{,}960 \text{ miles}$$

T12

HOW HIPPARCHUS MEASURED THE DISTANCE FROM THE EARTH TO THE MOON - A FIRST. (150 B.C.)



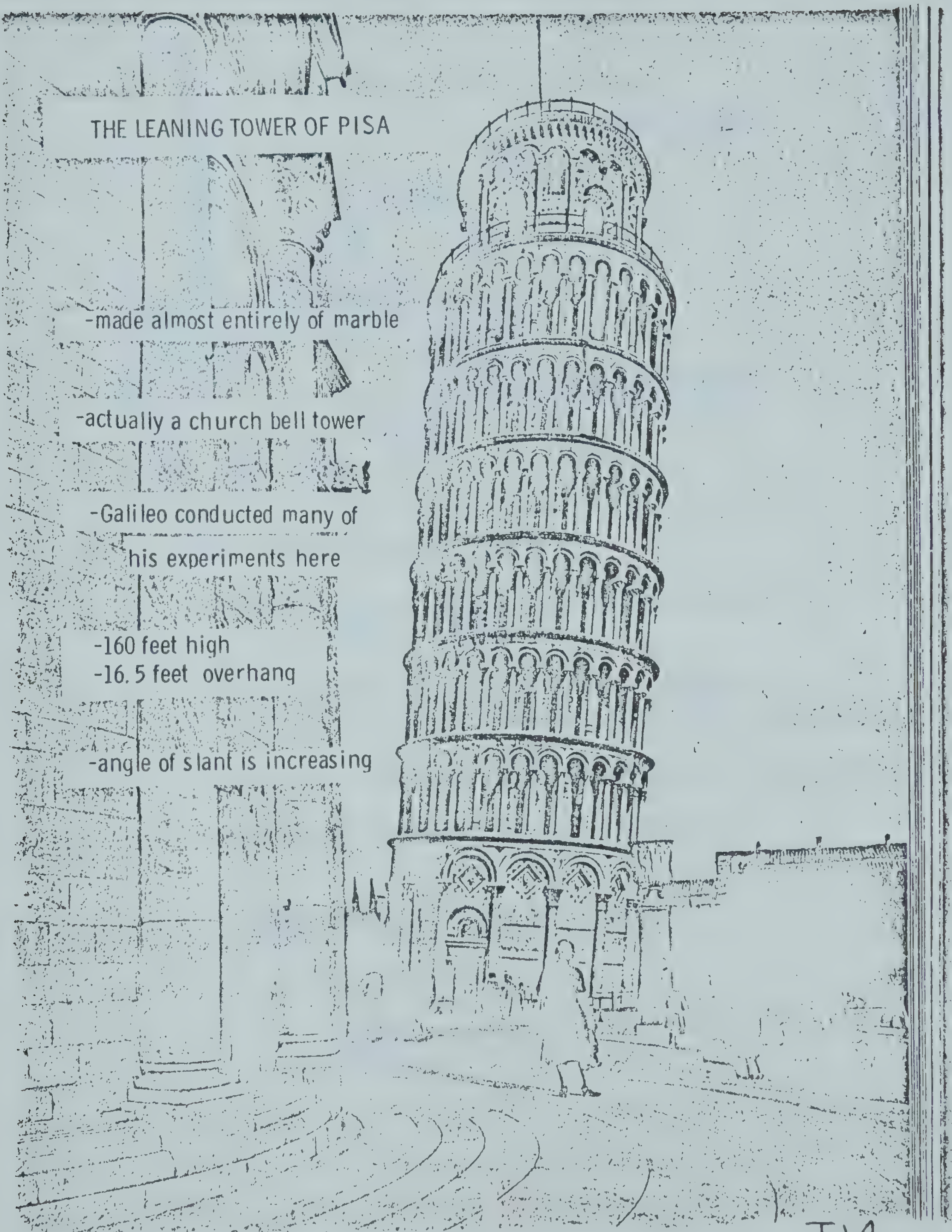
Method:

1. Find the radius of the earth (remember Eratosthenes measured the circumference).

2. Since C and D are points on the earth it was possible to find the distance between them, and hence angle CAD. How?

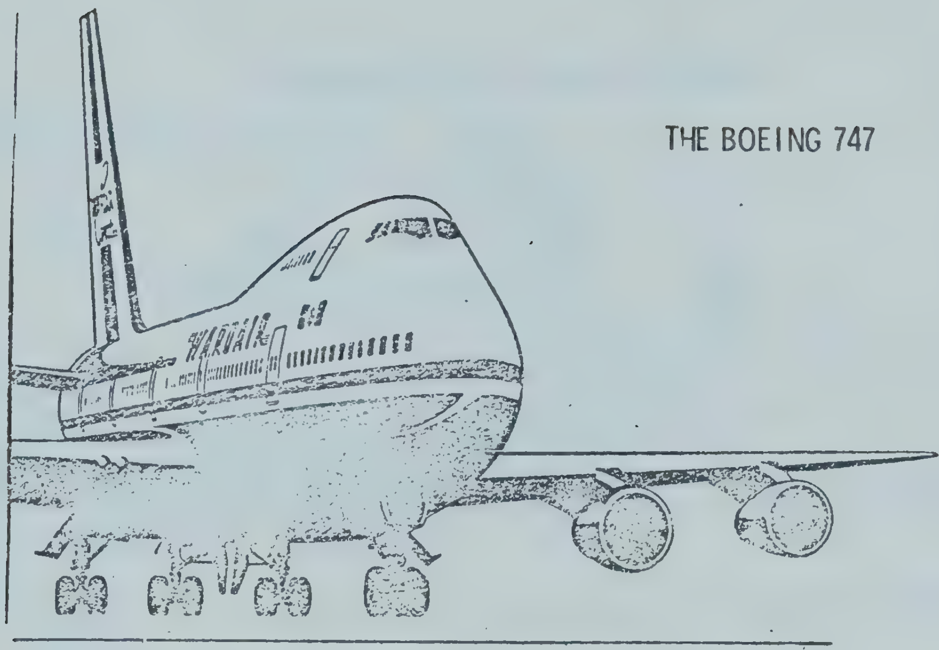
3. Use trigonometry to find AB.

4. What is the distance from the surface of the earth to the surface of the moon?


THE LEANING TOWER OF PISA
-made almost entirely of marble
-actually a church bell tower
-Galileo conducted many of his experiments here
-160 feet high
-16.5 feet overhang
-angle of slant is increasing

T14

THE BOEING 747



GROSS WEIGHT (LOADED) - 365 tons

CLIMB RATE - requires 40 minutes at 300 mph to reach cruise altitude of 31,000 feet

CRUISE INFORMATION - maximum altitude - 43,000 feet
- maximum nonstop distance - 6,000 miles

DESCENT - from 31,000 feet requires 100 nautical miles and 16 minutes

FUEL CONSUMPTION - 25,000 lbs/hour, climb or cruise

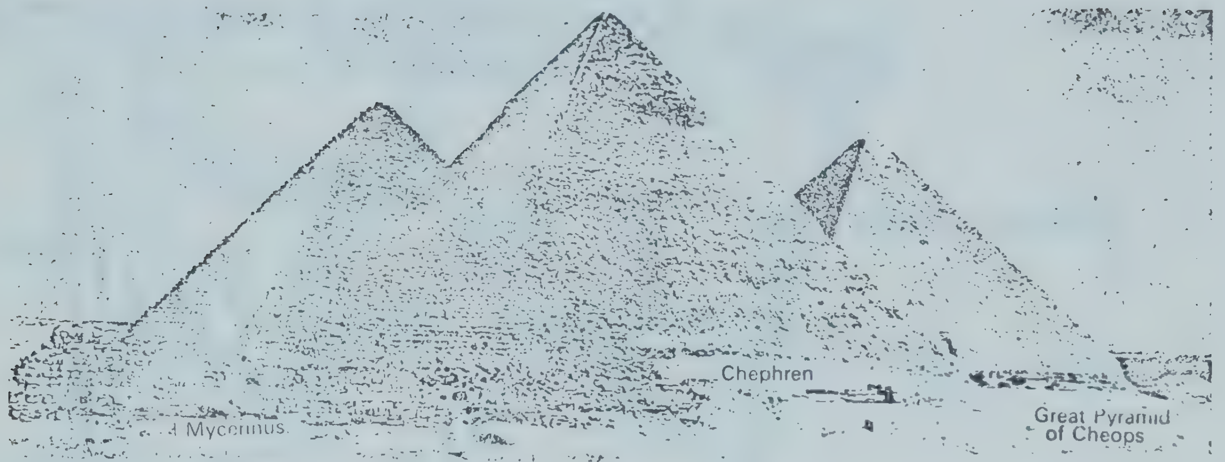
NON-STOP FLIGHTS

to **LONDON** every Friday via Wardair 747 Jumbo jet;

to **AMSTERDAM** via Wardair 707;

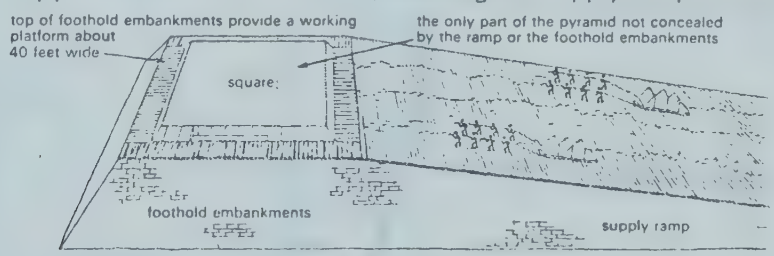
T15

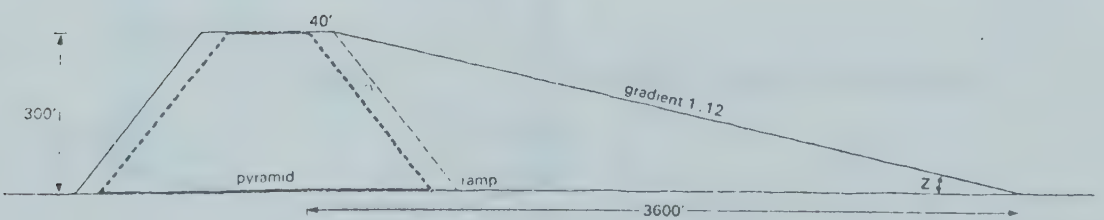
PYRAMIDS AND PYRAMID BUILDERS



- built along the Nile by Egyptian Kings as personal tombs and permanent memorials
- some are 4,000 years old
- largest are 400 feet high
- indicate exceptional engineering and mathematical skill on part of Egyptians
- required up to 30 years to construct
- well preserved because of dry climate

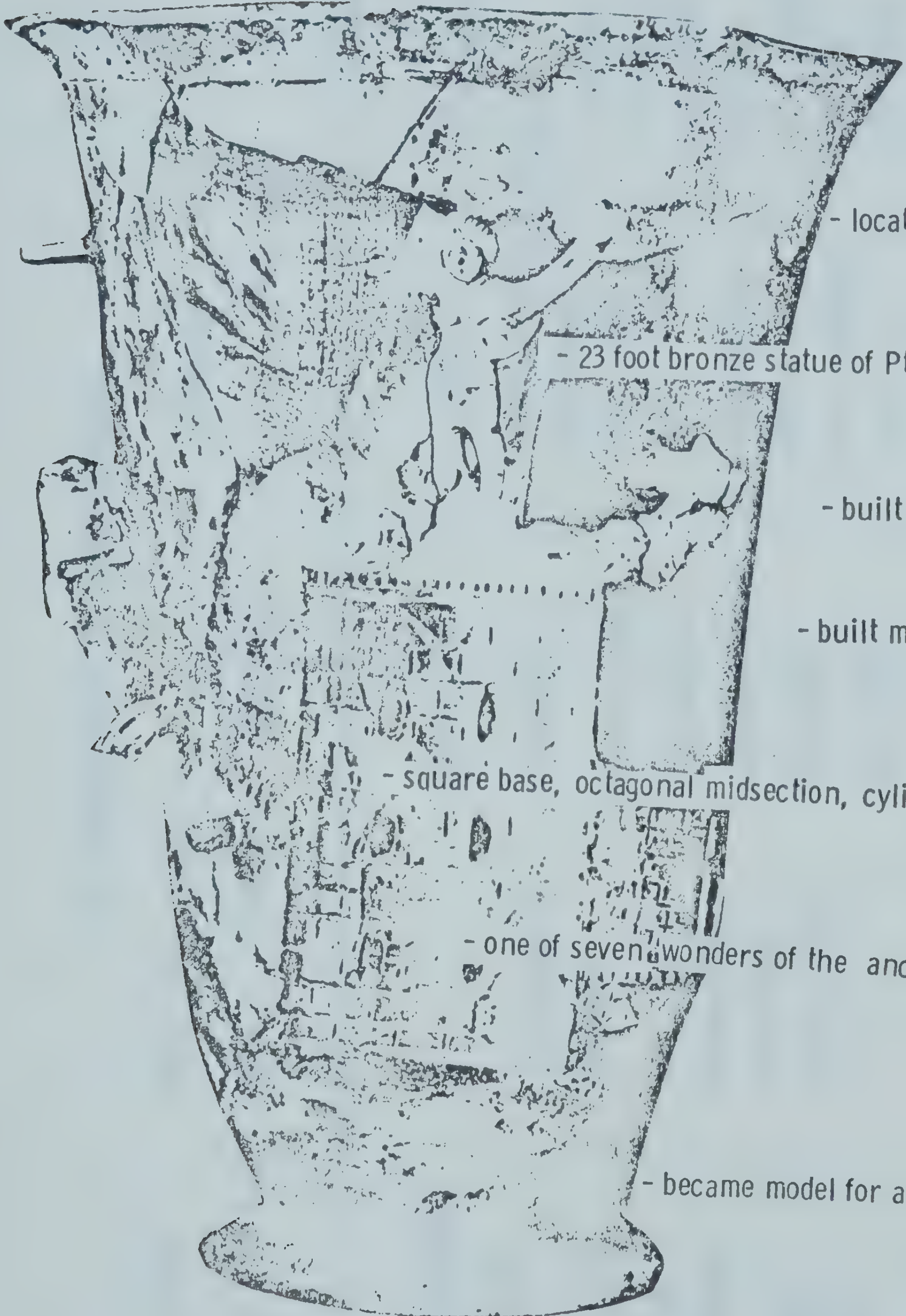
A pyramid under construction, showing the supply ramp





T16

THE PHAROS LIGHTHOUSE



THE DEVELOPMENT OF TRIGONOMETRY

GREEK	INDIA AND ARABIA	EUROPE		
150 B.C - 150 A.D.	500 A.D. - 1000 A.D.	1400 -1500	1550 - 1650	
HIPPARCHUS PTOLEMY		RHETICUS	VIÉTA	DÉSCARTES
- astronomers -fathers of trigonometry -first table of sines -charted over 1000 stars	-first tables of tangents -tables written as decimals for first time	-German -hired human computers to evaluate 10 place trig tables of all ratios	-French mathematicians -recognized trig ratios as functions -stressed the algebra of trig functions -first trig equations appear.	-invented rectangular coordinate system -first graph of trig functions appear (1635)

-developments in trigonometry were primarily related to investigations in astronomy up to about 1600

T18

APPENDIX 3

LESSON GUIDES AND STUDENT WORKSHEETS

LESSON 1

PURPOSE:

To introduce the topic of trigonometry and to review some of the necessary prerequisite mathematics.

OBJECTIVES:

At the conclusion of the lesson the students will be able to:

1. -state the two primary areas of application of trigonometry.

2. -state the Pythagorean theorem and solve elementary problems involving its use.

*3. -indicate the nature of the Pythagorean school and the major mathematical contribution made by Pythagoras.

4. -demonstrate that in similar right triangles, the ratio of corresponding sides is equal.

LESSON OUTLINE AND CLASSROOM PROCEDURE

INTRODUCTION:

1. Brief introduction to the topic stressing the two major areas where trigonometric methods find application in the real world namely, measurement and periodicity.

   a. Measurement:

      - navigation; air and sea
      - surveying, engineering (give examples)
      - astronomy

      *astronomy: give a brief (2-4 min.) talk on the contributions of Hipparchus and Ptolemy to the origins of trigonometry. See attached summary for their major contributions. Use overhead projectuals numbered T6 and T7.

   b. Periodicity:

      Trigonometry provides the mathematical model for most periodic events in our world.

- vibrating objects (springs, air columns, strings, membranes)
- all wave motion, including electricity, sound, water, light
- sonar, radar, short wave
- motion of a piston in a cylinder
- heartbeats, tides

2. Summary - because of its obvious usefulness, trigonometry is an important part of mathematics and worthy of study. The word trigonometry comes from three Greek words: tri - three, gonia - angle, and metria - measurement. Trigonometry is initially, the study of the relationships between the sides and angles of triangles.

REVIEW OF THE PYTHAGOREAN THEOREM

Since the study of trigonometry involves right triangles we begin by briefly reviewing the Pythagorean Theorem.

1. Pythagorean Theorem:

   a. Verbal statement of the theorem.
   b. Diagrammatic representation of the theorem.
   c. Algebraic equation.

   *Pythagoras and His School: give a brief (3-5 min.) talk on the history of what is now called the Pythagorean theorem, as well as the role played by Pythagoras and his school in the proof of the theorem. See attached summary and use projectuals numbered T1 and T5.

2. Example Problems on the Pythagorean Theorem

   Hand out worksheet 1 and do the first example on the assignment sheet with the class.

RATIOS OF CORRESPONDING SIDES IN SIMILAR RIGHT TRIANGLES

1. Introduction: In a given right triangle, if we know two sides we can find the other side. The questions we wish to ask now are (a) if we know two sides can we find the angles and (b) if we know a side and an angle can we find the other sides? These are questions that we answer in the study of trigonometry.

2. Necessary Terminology:

   a. Relative to a given angle students must be able to

identify the side <u>opposite</u> the given angle, the side <u>adjacent</u> to the given angle, and the <u>hypotenuse</u>.

b. Illustrate by blackboard examples.

c. Brief drill using 3 blackboard examples to identify the opposite, adjacent and hypotenuse relative to a certain angle.

3. <u>In-Class Worksheet on Ratios of Corresponding Sides in Similar Triangles</u>:

Explain the chart and what is required. Do the calculations for row 1 of the chart. Leave radicals unevaluated.

4. <u>Assignment</u>:

Complete all questions on Worksheet 1.

NOTES ON HIPPARCHUS AND PTOLEMY

HIPPARCHUS - 150 B.C.

- outstanding Greek astronomer
- worked in Alexandra, northern Egypt
- called the "Father of Trigonometry"
- wrote several books on astronomy - none exist today
- catalogued over 800 stars

PTOLEMY - 150 A.D.

- great Greek astronomer from Alexandria
- built upon the work of Hipparchus
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- his work unparalleled for 1000 years (until Copernicus)
- catalogued over 1000 stars

<u>Note</u>:

Trigonometry began as a response to the practical needs of astronomers, the need to measure the universe and the distance between the stars. At one time "mathematics" meant "astronomer."

NOTES ON PYTHAGORAS AND HIS CULT

PYTHAGORAS - 580 B.C. to 500 B.C.

- outstanding Greek mathematician and mystic
- travelled in Egypt, India and Babylon where he no doubt learned much mathematics
- founded a school and brotherhood in southern Italy known as "The Order of the Pythagoreans"
- a semireligious cult developed which became very secretive and mystical - had a secret emblem indicating membership in the cult (the pentagram)
- Pythagoras is given credit for the proof of the "Pythagorean Theorem" even though the theorem had been used by the Babylonians at least 1000 years earlier.

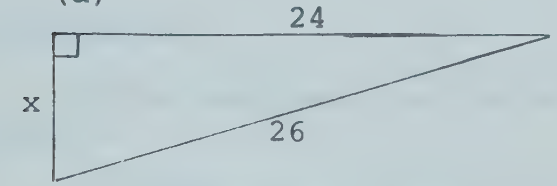
WORKSHEET 1

TRIGONOMETRY MATH 23

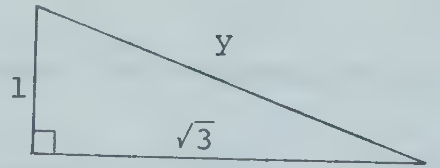
1. PYTHAGOREAN THEOREM

In each of the following right triangles, find the measure of the unknown side.

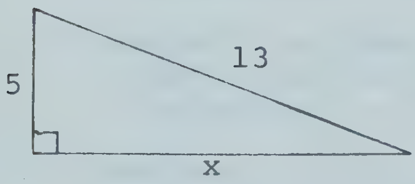
(a)



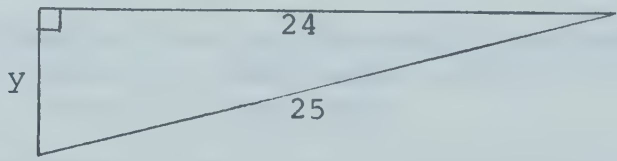
(b)



(c)

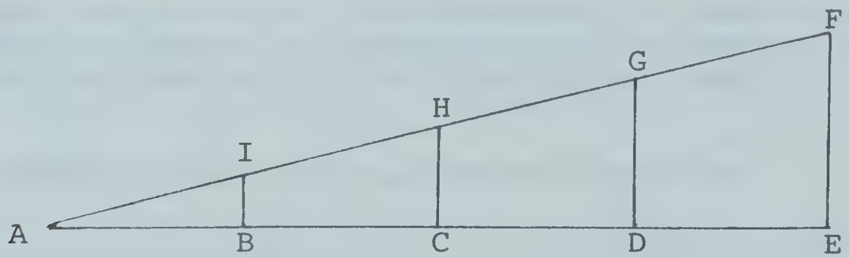


(d)



2. RATIOS OF SIDES IN SIMILAR TRIANGLES

a. Measure angle A and label it on the diagram below.
b. Calculate the lengths of AI, AH, AG and AF.
c. Complete the chart below, and then make as many observations as you can regarding your findings.



	$\frac{OPP}{HYP}$	$\frac{ADJ}{HYP}$	$\frac{OPP}{ADJ}$
Δ ABI			
Δ ACH			
Δ ADG			
Δ AEF			

LESSON 2

PURPOSE:

To introduce the primary trigonometric ratios of an angle, how to evaluate them, and to show that each ratio is a function of angle measure.

OBJECTIVES:

At the conclusion of this lesson the students will be able to:

1. -define the primary trigonometric ratios of a given angle in terms of the opposite side, the adjacent side and the hypotenuse.

2. -explain why the values of the trigonometric ratios depend only on the measure of the angle and not on the lengths of the "arms" of the angle.

3. -estimate any trigonometric ratio of any acute angle by drawing an appropriate diagram and making the necessary measurements and calculations.

4. -use trigonometric tables to find the value of any trigonometric ratio of any given acute angle measured in degrees.

*5. -trace the evolution of trigonometric ratio values from the tables of Hipparchus and Ptolemy to the computers and pocket calculators of today.

LESSON OUTLINE AND CLASSROOM PROCEDURE

PRIMARY TRIGONOMETRIC RATIOS

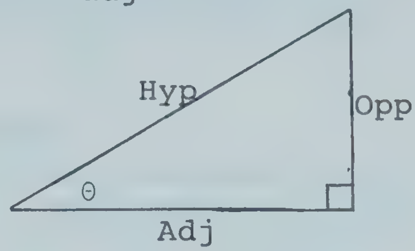
1. Worksheet 1

On the overhead transparency of Worksheet 1 fill in the chart and make appropriate observations, that is, the ratios of corresponding sides in similar right triangles are equal. Write this conclusion on the blackboard. (Students should record this summary on their worksheets.)

2. Defining the Primary Trigonometric Ratios

   (a) Because of the observations made in Worksheet 1 we define the three primary trigonometric ratios as follows:

   $$\sin\theta = \frac{opp}{hyp};\ \cos\theta = \frac{adj}{hyp};\ \tan\theta = \frac{opp}{adj}$$



   (b) Introduce the mnemonic SOH CAH TOA and explain how it is used.

   (c) Examples: do questions 1(a) and (b) of Worksheet 2.

TO SHOW TRIGONOMETRIC RATIOS ARE A FUNCTION OF ANGLE MEASURE

1. In-Class Worksheet

   (a) In order to discover that the trigonometric ratios vary with θ, have students complete question 2 of Worksheet 2. It may be wise to divide the responsibility for calculationg among various students or groups because of the time factor.

   (b) When the calculations are completed, fill in the results on the overhead transparency of Worksheet 2. Have students then fill in their charts from the projectual.

   (c) Make appropriate observations and summarize the findings in the "rectangle" on the projectual. The function idea of input-output might be helpful here.

HOW TO FIND TRIGONOMETRIC RATIOS OF ANY ACUTE ANGLE

1. Students should now be able to approximate the value of any trigonometric ratio of any acute angle by appropriate construction, measurement and calculation (without using tables). Explain.

2. Trigonometric Tables:

   (a) Explain how to use trig tables and why they are a valuable aid. Provide adequate in-class experience on the use of tables.

   (b) Compare the values of the trig ratios calculated in question 2 of Worksheet 2 with the corresponding values found in the tables. Explain the discrepancy.

   *Evolution of Trigonometric Tables

   Using projectual no. T8 as a guide, discuss briefly the origin and main stages of development of trigonometric tables, including the methods of calculation. Demonstrate the speed with which any trig ratio of any angle may be calculated using a scientific minicalculator. Show a computer output of trig tables and compare the time required for calculation with the time required by Rheticus (16th Century).

   Point out the impact that mathematics has had on the design and development of calculators and computers, and the impact that they have on our modern society.

3. Assignment

   Complete all questions on Worksheet 2.

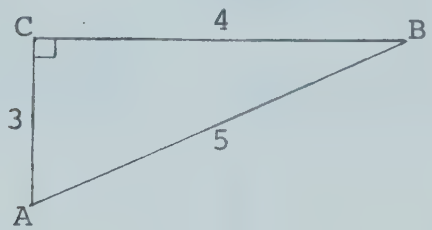
WORKSHEET 2

TRIGONOMETRY MATH 23

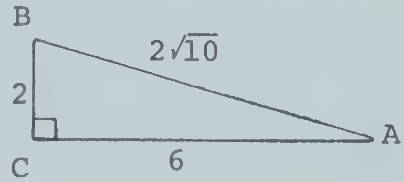
1. PRIMARY TRIGONOMETRIC RATIOS

1. For each of the following right triangles, find the sin, cos and tan of Angle A and also Angle B. Leave your answers in fractional form.

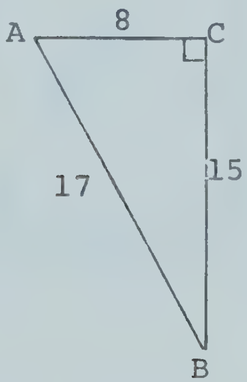
(a)



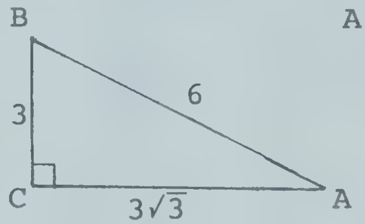
(b)



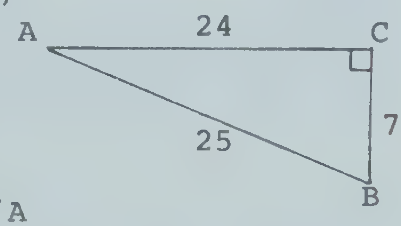
(c)



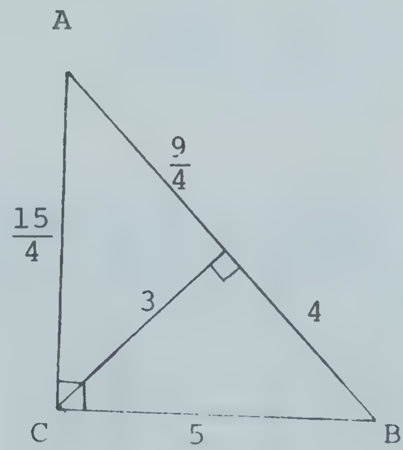
(d)



(e)

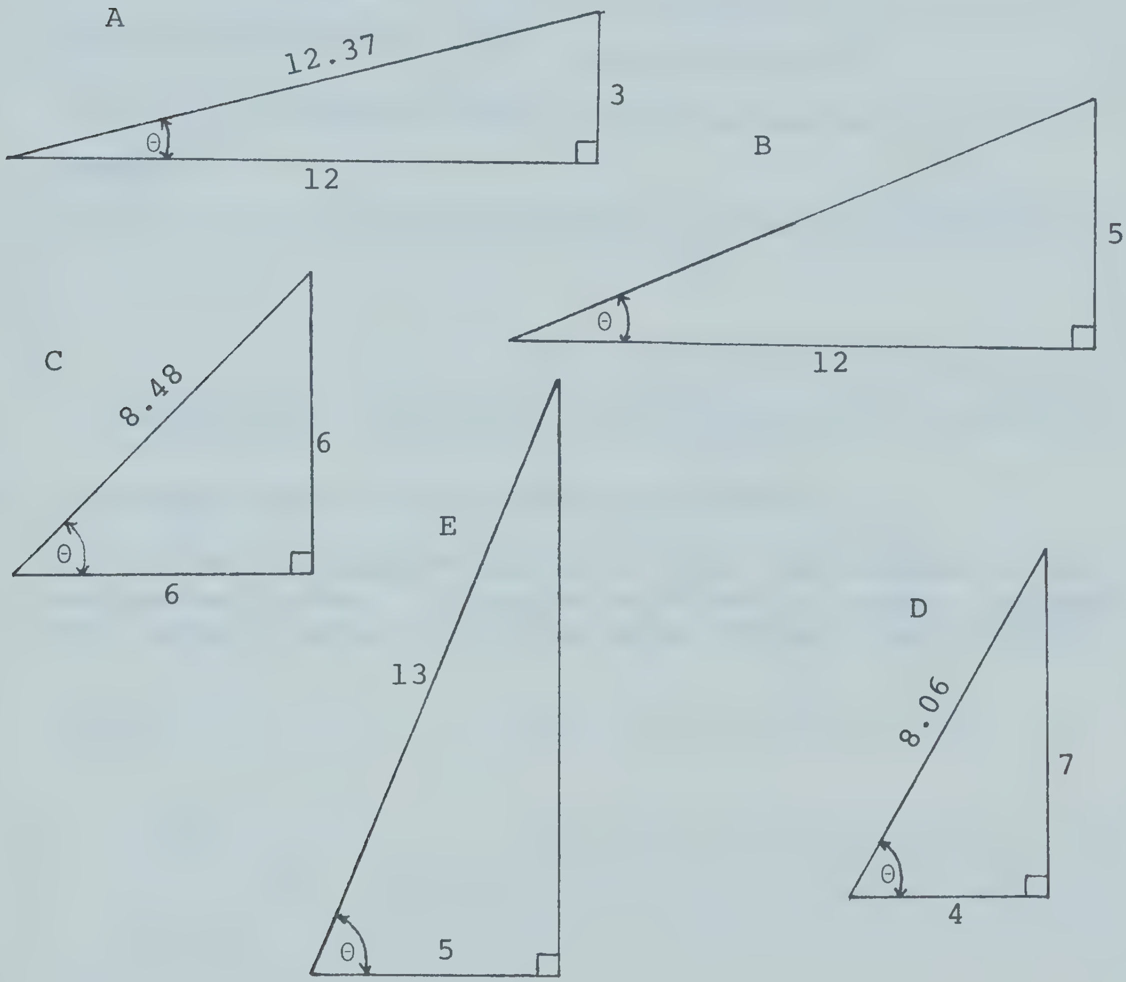


2. Using the diagram to the right, find the primary trigonometric ratios of
(a) $\angle$ BCD (b) $\angle$ ACD
(c) $\angle$ A (d) $\angle$ B



2. DO TRIG RATIOS VARY WITH THE MEASURE OF THE ANGLE?

In each of the following triangles measure the indicated angle (to the nearest degree) and label it. Calculate the trigonometric ratios for these angles in each of the triangles. Record your findings in <u>decimal form</u> in the chart.



Δ	θ	opp	adj	hyp	SIN θ	COS θ	TAN θ
A							
B							
C							
D							
E							

When you have completed the chart answer the following questions which relate to your findings.

1. As the measure of the angle varies, does the measure of the trigonometric ratios vary?

2. From the chart (which involves acute angles only) we have:

   (a) as θ increases, $\sin \theta$ ________________.

   (b) as θ increases, $\cos \theta$ ________________.

   (c) as θ increases, $\tan \theta$ ________________.

   SUMMARY

3. A PROBLEM THAT IS NOW WITHIN YOUR GRASP

   To "solve a triangle" means to find the measure of all angles and sides. Solve triangle ABC if angle B equals 55°, angle C equals 90°, and side AB has length 36.5.

LESSONS 3 AND 4

PURPOSE:

To solve right triangles using trigonometric methods.

OBJECTIVES:

At the conclusion of these two lessons the students should be able to:

1. -Solve a right triangle given one angle and one side.

2. -Solve problems that occur in the real-world whose solution requires a knowledge of trigonometric methods.

*3. -Name two mathematicians who contributed to the origin and development of trigonometry, and one historical problem of significance whose solution depended on trigonometric methods.

LESSON OUTLINES AND CLASSROOM PROCEDURES

LESSON 3

1. SOLVING RIGHT TRIANGLES AND APPLICATIONS OF TRIGONOMETRY

   a. Worksheet 2:
   Take up and discuss any problems encountered in the assignment from Worksheet 2. Pay particular attention to Problem 3 since it introduces the ideas central to this lesson.

   b. Worksheet 3 and 4:
   1. Hand out Worksheet 3 and 4 and do question 1a, 1b and 2a on the board as example problems. Note that separate worksheets have been devised for treatments R (regular) and C (cultural) and that question 2a is different on these separate worksheets. Explain the meaning of "angle of elevation" and "angle of depression."

      * Emphasize the practical importance of the methods of trigonometry to solve real-world problems of great importance to the wellbeing of the student and his society, i.e., engineering,

surveying, air navigation, sea navigation and so on.

2. ASSIGNMENT

The assignment is left to the discretion of the instructor, nevertheless do not assign any questions from question 3 of Worksheet 3 and 4.

LESSON 4

1. APPLICATIONS OF TRIGONOMETRY (CONTINUED)

   a. Worksheet 3 and 4:
   Take up and discuss problems encountered in the assignment of last class.

   b. Have class R proceed with the remaining questions on Worksheet 3 and 4.
   * In class C discuss the historical significance of the ancient city of Alexandria in the field of mathematics (trigonometry in particular). See the information provided and use projectuals numbered T1 and T10. Then discuss Eratosthenes and the method he used to determine the circumference of the earth. See the information provided and use projectuals numbered T11 and T12.
   Then discuss the method used by Hipparchus to calculate the distance from the earth to the moon. See the information provided and use projectual numbered T13.
   Then have the students proceed with the remaining questions on Worksheets 3 and 4.

   c. Test:
   Remind the students that a full-period test will be given next class-time covering all materials studied to date (in this unit). In particular students will be tested on the mathematics outlined in the individual lesson objectives.

THE CITY OF ALEXANDRIA

About 325 B.C., Alexander the Great conquered all Greece, Egypt, and the Near East. He then founded and built the great city of Alexandria, on the south shore of the Mediterranean Sea, just west of the mouth of the Nile. It was built as the capital city of the new empire and was located at the junction of Europe, Asia and Africa. It became the commercial, intellectual and cultural centre of the entire ancient world. Alexandrian traders imported knowledge that had been acquired all over the world. Soon large libraries, laboratories, and museums were built and the greatest minds of the time were assembled to study.

Such great names in mathematics and science as Archimedes, Euclid, Appolonius, Eratosthenes, Hipparchus, Claudius, Ptolemy and Heron all studied and worked in Alexandria.

By about 600 AD this majestic city with its treasures had been completely destroyed by invading armies. A new and modern city of Alexandria exists today on the same sight as the ancient city.

TWO HISTORICALLY SIGNIFICANT PROBLEMS

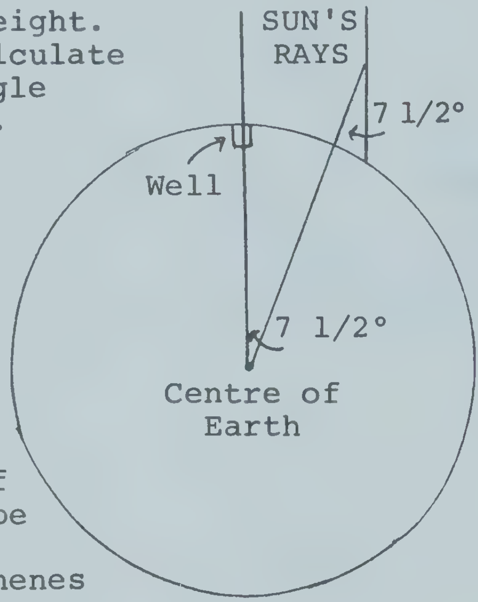
1. HOW ERATOSTHENES FOUND THE CIRCUMFERENCE OF THE EARTH - A FIRST!

   a. Discussion concerning Eratosthenes (230 B.C.)
      - lived in Athens many years
      - invited to study at Alexandria and serve as the

chief librarian and tutor to the king's son.
- extremely gifted individual
- athlete, poet, astronomer, geographer, historian, mathematician

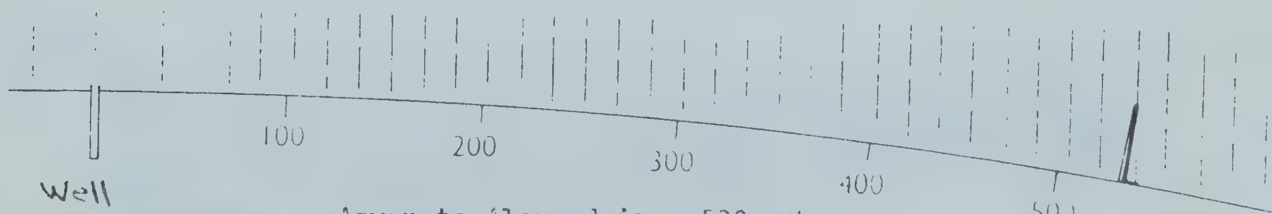
b. How Eratosthenes calculated the circumference and the radius of the earth 2000 years ago.

He knew that at noon on a certain day of the year, the rays of the sun were reflected from the water in a well near Aswan on the Nile (the sun would cast no shadows at that time). At the same time at Alexandria, 520 miles to the north, Eratosthenes measured the length of the shadow cast by a pillar of known height. From this he was able to calculate (using trig) the central angle as indicated on the diagram. Since Aswan and Alexandria were on the same meridian he was then able to calculate the circumference of the earth by setting up a proportion as follows:



$\frac{7.5°}{520} = \frac{360°}{C} \Rightarrow C = 24{,}960$ miles

Since $C = 2\pi 4$, the radius of the earth was estimated to be 3,974 miles.
The calculations of Eratosthenes were amazingly accurate.



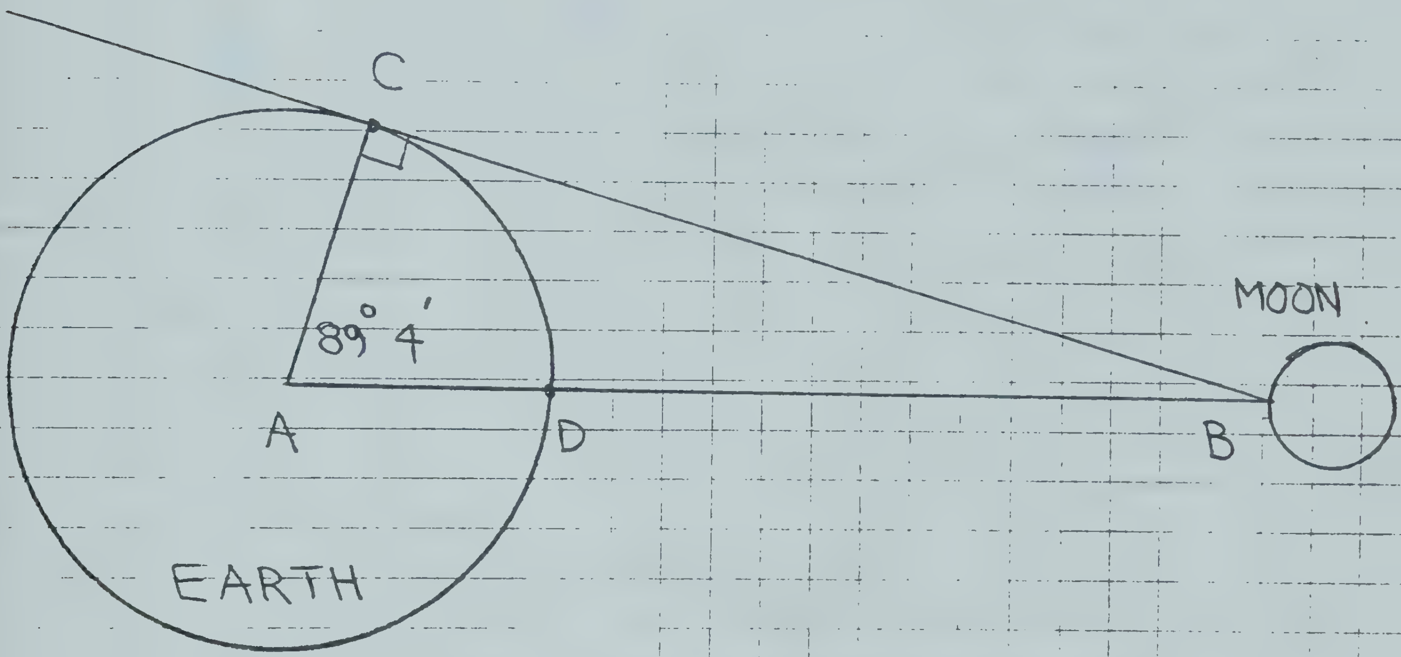
Aswan to Alexandria – 520 miles

2. HOW HIPPARCHUS (150 B.C.) FOUND THE DISTANCE FROM THE EARTH TO THE MOON - ANOTHER FIRST!

How he did it:
Suppose at a certain time the moon is observed directly over a certain point, D on the earth, and at the same

instant another observer at point C watches the moon rise on the horizon. Since he could find the distance between C and D, it was a simple matter to calculate $\angle A$, since $\frac{\angle A}{\text{Arc CD}} = \frac{360°}{\text{Circumference of earth}}$. A turns out to be 89° 4'.

Since ΔABC is a right triangle and AC is the radius of the earth it was a simple matter to calculate AB as approximately 238,000 miles.



NOTE:

1. Other Alexandrians determined the size of the moon and the distance to the sun. These were the first daring voyages of man's mind into the great expanse of outer-space. They represented an epochal advance in man's knowledge of the universe about him.

2. Note the significance of the problems solved by these relatively simple methods of trigonometry. Trigonometry has always aided man to understand and explain his environment more completely.

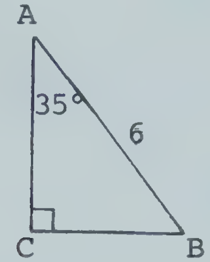
WORKSHEET 3 AND 4 (R)

TRIGONOMETRY MATH 23

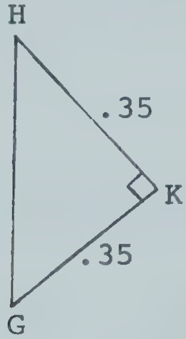
1. SOLVING RIGHT TRIANGLES

Solve each of the following right triangles.

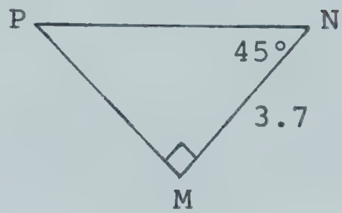
(a)



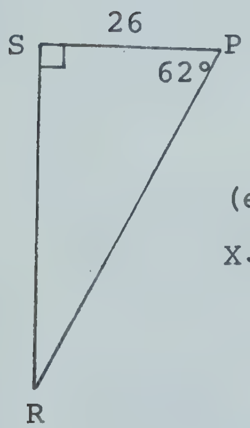
(b)



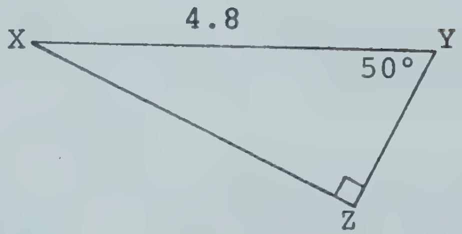
(c)



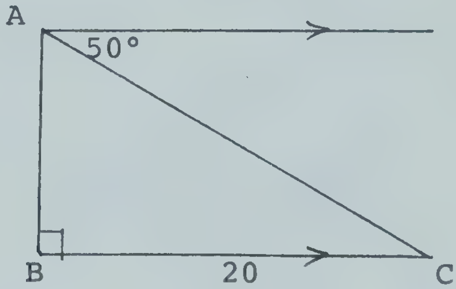
(d)



(e)



(f)



2. APPLICATIONS OF TRIGONOMETRY

a. The angle of elevation to the top of a building from a point 1,000 feet from its foot is 35°. Find the height of the building.

b. From the top of a lighthouse 90 feet high, the angle of depression of a ship out on the lake is 5°. How far is the ship from the base of the lighthouse?

c. An aircraft leaves the ground with an angle of climb of 15°. Its speed is 120 mph. If the plane levels off at the end of a 10 minute climb, at what altitude is it cruising?

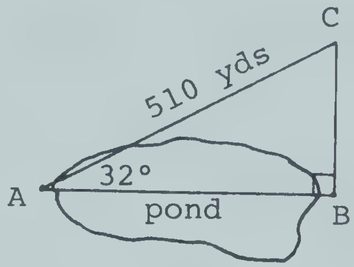
d. The vertical angle of an isosceles triangle measures 58° and its base is 10 inches. Find its altitude and area.

e. A weather balloon is released in a 15 mph wind at ground level and after 20 minutes is observed at an angle of elevation of 18°.
   (1) How high is the balloon after 20 minutes?
   (2) How far is the balloon from the release point?
   (3) What was the ascent rate of the balloon in miles per hour?

3. MORE APPLICATIONS

a. From the top of a mountain the angle of depression to a point A on the level of the base is 27°. If AB = 4498 feet find the height of the mountain.

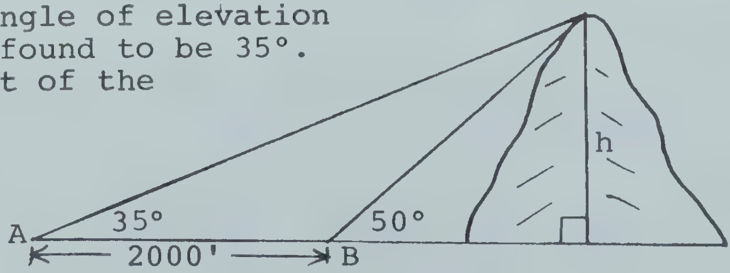
b. To find the distance from point A to point B across a pond, a surveyor makes the measurements shown in the diagram. Find AB.



c. The angle of elevation of a road is known to be 10°. How high will the road rise for a distance of 480 feet measured along the road (to the nearest foot)?

d. A ladder 32 feet long just reaches the eaves of a building when the ladder makes an angle of 75° with the ground. If the base of the ladder is placed an additional 3 feet away from the wall of the building, where will the top of the ladder now be with relation to the eaves?

e. From a point B near the base of a mountain the angle of elevation to the peak is found to be 50°. At a point A which is 2000 feet directly away from the mountain (and point B) the angle of elevation to the top is found to be 35°. Find the height of the mountain.



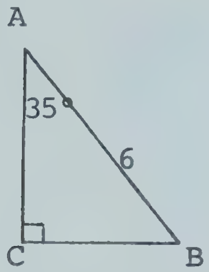
WORKSHEET 3 AND 4 (C)

TRIGONOMETRY MATH 23

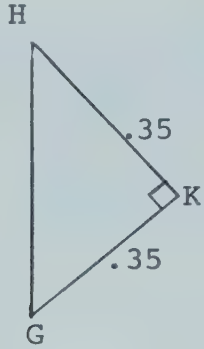
1. SOLVING RIGHT TRIANGLES

Solve each of the following right triangles.

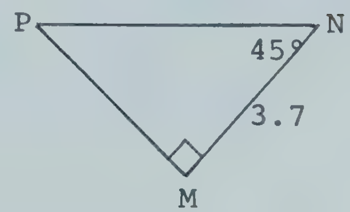
(a)



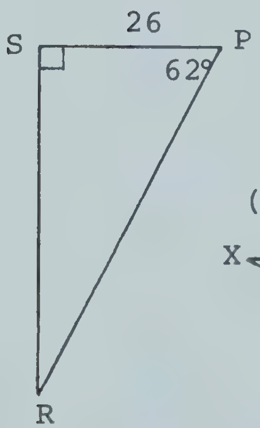
(b)



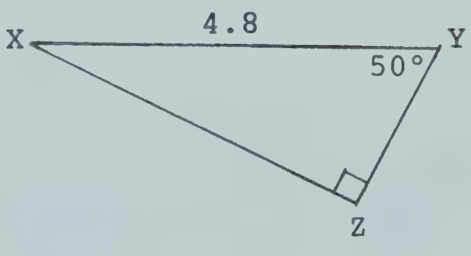
(c)



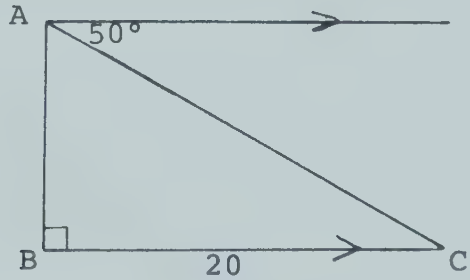
(d)



(e)



(f)



2. APPLICATIONS OF TRIGONOMETRY

a. At a distance of 4.78 miles from the base of Edmonton's Alberta Telephone Tower, the angle of elevation to the helicopter landing pad on the top of the building is exactly 1°. Calculate the height of the tower to the nearest foot.

On a clear day, from any window on Vista 33 (the thirty third floor of the tower) one can see approximately 28 miles. What is the total number of square miles visible from Vista 33.

b. From the top of a lighthouse 90 feet high, the angle of depression of a ship out on the lake is 5°. How far is the ship from the base of the lighthouse?

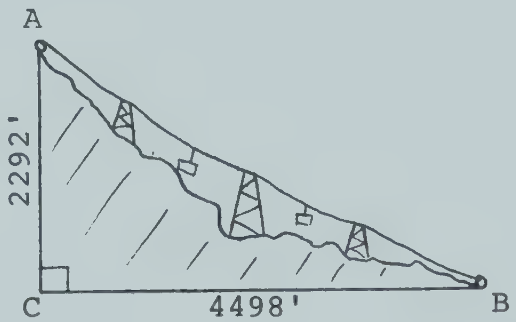
c. A light aircraft leaves the runway at Edmonton's industrial airport and proceeds to climb at an angle of elevation of 15°. Its speed is 120 mph. If the plan levels off at the end of a 10 minute climb, at what altitude is it cruising?

d. The vertical angle of an isosceles triangle measures 58° and its base is 10 inches. Find its altitude and area.

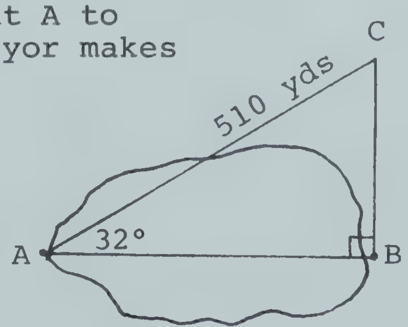
e. A weather balloon was released at the International Airport in a 15 mph wind. After 20 minutes it was observed at an angle of elevation of 18°.
   (1) How high was the balloon after 20 minutes?
   (2) What was the linear distance of the balloon from the release point?
   (3) What was the ascent rate of the balloon in miles per hour?

3. MORE APPLICATIONS

a. The Sulphur Mountain Gondola Lift at Banff rises at an angle of 27° to the horizontal. The distance from C to B (see diagram) is 4498'. Find the vertical distance through which a passenger rises in going from the base to the summit of Sulphur Mountain. Air temperature decreases 2 celcius degrees per 1000 foot increase in altitude. Find the temperature at the summit of the mountain when the temperature at the base is 20° C.



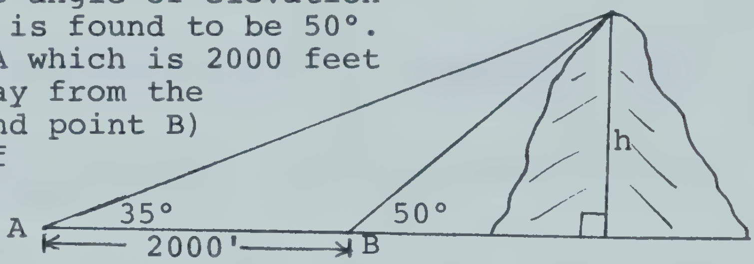
b. To find the distance from point A to point B across a pond, a surveyor makes the measurements shown in the diagram. Find the width of the pond (length AB).



c. The angle of elevation of a road is known to be 10°. How high will the road rise for a distance of 480 feet measured along the road (to the nearest foot)?

d. A ladder 32 feet long just reaches the eaves of a building when the ladder makes an angle of 75° with the ground. If the base of the ladder is placed an additional 3 feet away from the wall of the building, where will the top of the ladder now be with relation to the eaves?

e. From a point B near the base of a mountain the angle of elevation to the peak is found to be 50°. At a point A which is 2000 feet directly away from the mountain (and point B) the angle of elevation to the top is found to be 35°. Find the height of the mountain.



LESSONS 5 AND 6

PURPOSE:

To solve right triangles given any two sides.

OBJECTIVES:

At the conclusion of this lesson the students should be able to:

1. -find the angle given any primary trigonometric ratio of an angle.

2. -solve a right triangle given any two sides of the triangle.

3. -solve problems that occur in the real world whose solution involves the solving of a triangle when two of the sides are known.

*4. -state two ways that the methods of trigonometry find application in modern society.

LESSON OUTLINE AND CLASSROOM PROCEDURE

LESSON 5

1. <u>Finding the Angle Given one of its Trigonometric Ratios</u>:
   Demonstrate how trigonometric tables may be used to find the angle whose trigonometric ratio is given. Use the tables in the text and do several orally.

2. <u>Solving Right Triangles Given any Two Sides</u>:
   Distribute Worksheet 5 and 6. Note that <u>separate worksheets</u> have been devised for treatments R and C.

   Work question 2(a) on the blackboard.

3. <u>Applications of Right Triangles</u>:
   Work question 3(a) on the blackboard. Note that these questions are different on the respective worksheets.
   *Use transparency no. T14 (leaning tower of Pisa) to introduce the problem.
   Transparency T15 is to be used in conjunction with problem 3(c).

4. Assignment:
   Have students begin work on Worksheets 5 and 6 and do as many problems as seems reasonable.

LESSON 6

1. Discuss any difficulties encountered in the assignment.

2. Complete Worksheets 5 and 6.
   *Use transparencies T16 and T17 in conjunction with problems 4(a) and 4(b).

WORKSHEET 5 AND 6 (R)

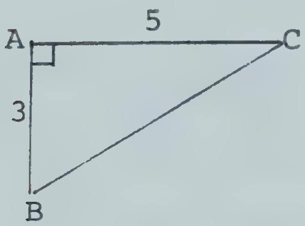
TRIGONOMETRY MATH 23

1. In each of the following questions the trigonometric ratio of an unknown angle A is given. Use trigonometric tables to find the measure of A to the nearest degree.

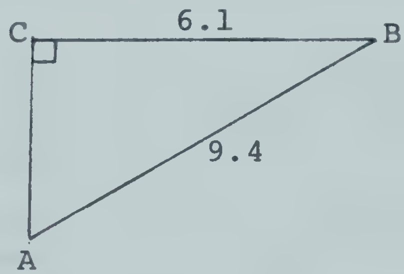
   a. sin A = .3255; A =
   b. cos A = .0174; A =
   c. tan A = 7.115; A =
   d. cos A = .4263; A =
   e. tan A = 1.8913; A =
   f. sin A = .9876; A =

2. Solve each of the following triangles:

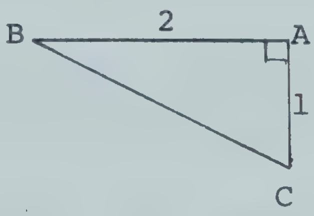
(a)



(b)



(c)



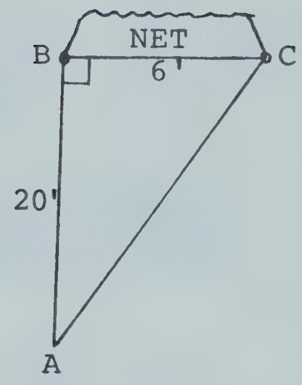
3. APPLICATIONS OF TRIGONOMETRY

   a. A ladder 24 feet long is placed against a vertical wall. What angle does the ladder make with the wall if the foot of the ladder is 5 feet from the base of the wall?

   b. A railroad has a gradient of 3%; that is, the track rises vertically 3 feet in a horizontal distance of 100 feet.
      (1) What is the tangent of the angle that the track makes with the horizontal?
      (2) What height will the track rise in a horizontal distance of one mile?

   c. A vertical pole 6 feet high casts a shadow 10.5 feet in length.
      (1) What is the tangent of the angle of elevation of the sun?
      (2) Determine the angle of elevation of the sun.

d. A hockey player is located at point A, 20 feet out from the post B of the goal. If the second post is labelled C, then angle ABC = 90°. If the width of the goal is 6 feet, within what angle must the player keep his shot in order to be "on the net"?



4. a. A road has a gradient of 1:12.
   (1) Calculate the angle of elevation of the road (to the nearest degree).
   (2) Calculate the distance one would need to travel along the road in order to change one's altitude by 300 feet.

   b. If from a boat 43.29 miles from the base of a lighthouse the angle of elevation to the top of the lighthouse is 6 minutes, find the height of the lighthouse.

   c. From the top of a cliff 240 feet high, the angle of depression of a buoy is 18°. Find the distance of the buoy from the cliff.

WORKSHEET 5 AND 6 (C)

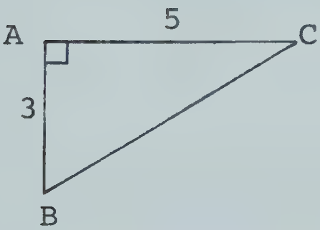
TRIGONOMETRY MATH 23

1. In each of the following questions the trigonometric ratio of an unknown angle A is given. Use trigonometric tables to find the measure of A to the nearest degree.

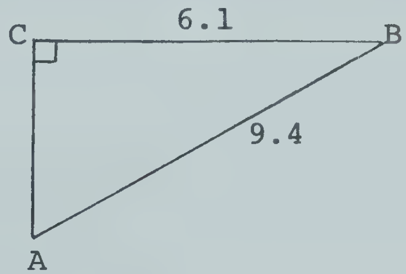
a. sin A = .3255; A =	d. cos A = .4263; A =
b. cos A = .0174; A =	e. tan A = 1.8913; A =
c. tan A = 7.115; A =	f. sin A = .9876; A =

2. Solve each of the following triangles:

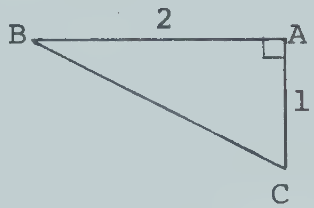
(a)



(b)

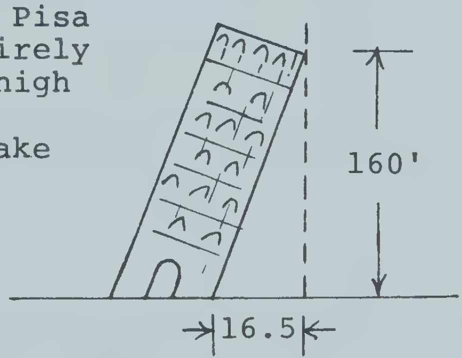


(c)



3. APPLICATIONS OF TRIGONOMETRY

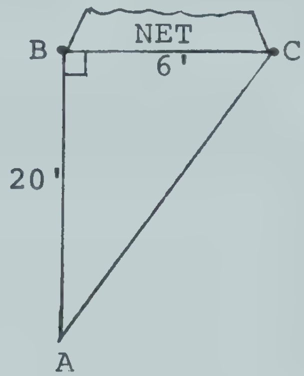
a. The famous leaning tower of Pisa in Italy is made almost entirely of marble. It is 160 feet high and overhangs by 16.5 feet. What angle does the tower make with the vertical?



b. The maximum gradient allowed on any new four-lane highway in the Province of Alberta (except in the National Parks) is 5%; that is, the road rises vertically 5 feet in a horizontal distance of 100 feet. Assuming a 5% gradient:
(1) What is the tangent of the angle that the road makes with the horizontal?
(2) What height will the road rise in a horizontal distance of one mile?

c. The world's largest passenger aircraft, the Boeing 747, flys from Edmonton's International Airport to points all over the world. This airplane requires 40 minutes (from takeoff) to reach their cruise altitude of 31,000 feet. If their climb speed is 300 mph, what is their angle of ascent above the horizontal during the climb? Since these planes burn 625 lbs. of fuel per minute, how many tons of fuel are required to reach their cruise altitude?

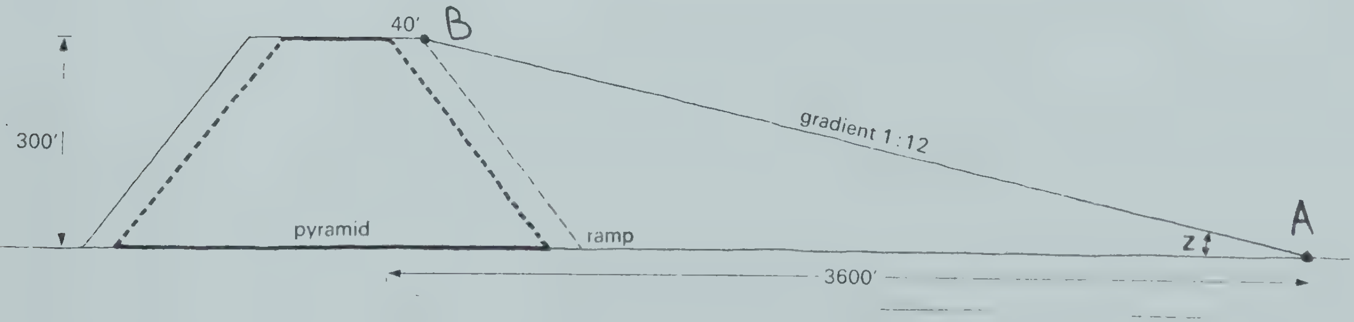
d. A hockey player is located at point A, 20 feet out from the post B of the goal. If the second post is labelled C, then Angle ABC = 90°. If the width of the goal is 6 feet, within what angle must the player keep his shot in order to be "on the net"?



4. PROBLEMS IN TRIGONOMETRY OF HISTORICAL INTEREST

a. PYRAMID RAMPS:

During the building of the pyramids a ramp was used to transport the stone blocks to the "top." This ramp was always kept at a gradient of 1:12.
(1) Calculate the angle of elevation of the ramp (to the nearest degree).
(2) Calculate the length of the ramp AB when the pyramid under construction was 300 feet high.



b. HEIGHT OF THE PHAROS LIGHTHOUSE:

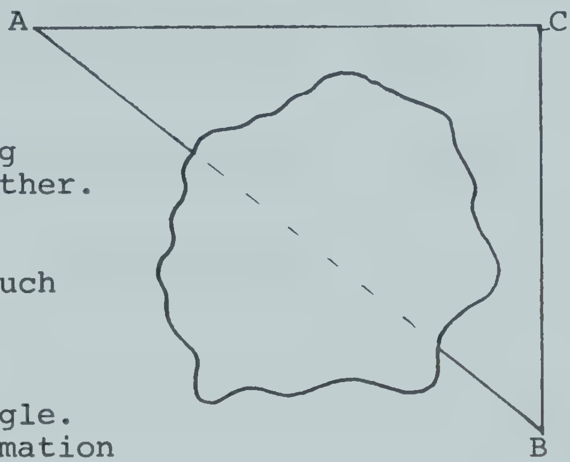
Around 275 BC in the city of Alexandria, King Ptolemy II built a massive lighthouse which became the model of all lighthouses constructed thereafter. It was called the Pharos. The lowest

section was square, the middle section octagonal and the uppermost section cylindrical in shape. It was constructed almost entirely of marble. The light on the top, probably a fire that was kept burning at night could be seen for miles by sailors on the Mediterranean.

If from a boat 43.29 miles from the base of the lighthouse the angle of elevation to the top of the lighthouse was 6 minutes, find the height of the lighthouse.

c. HERON OF ALEXANDRIA:

Heron (100 AD) was an engineer and mathematician whose genius left his contemporaries in awe (mainly because he knew some mathematics). He showed how to dig a tunnel through a mountain by beginning at either end and having the borings meet each other. He suggested choosing a point B on one side and point A on the other, such that both points are visible from a point C, with C chosen so that angle BCA is a right angle. From this limited information Heron suggested the job could be completed. Explain.



LESSONS 7 AND 8

PURPOSE:

To provide experiences whereby methods of trigonometry can be used to calculate heights indirectly.

OBJECTIVES:

At the conclusion of this lesson the students should be able to:

1. -measure lengths and angles using a tape measure and homemade clinometer.

2. -calculate the heights of inaccessible objects using only a tape measure, a clinometer, and the methods of trigonometry.

*3. -appreciate and recognize the importance of trigonometry in surveying and construction.

LESSON OUTLINE AND CLASSROOM PROCEDURE

LESSON 7

1. PRE-EXCURSION DISCUSSION

A half-period of class time should be spent on the following items:

- -outlining the purpose of the outdoor assignment
- -explaining the operation of the equipment to be used (tape and clinometer)
- -explain why the clinometer actually measures the angle of elevation (use a large blackboard diagram)
- -outline the assignment relative to what will be required of them, i.e., the data that they will be required to find.
- -stress the importance of accurate measurements.
- -explain assignment sheets. (the instructor is responsible for the design of these)

LESSON 8

ACTUAL OUTDOOR EXCURSION

-have students assemble in classroom
-distribute tapes and clinometers
-distribute assignment sheets
-proceed with assignment
-assigned calculations should be completed for the next class.

LESSON 9 (R)

PURPOSE:

To review the trigonometry studied to date and to draw graphs of the sine and cosine functions.

OBJECTIVES:

At the conclusion of this lesson the students should be able to:

1. -explain why trigonometric ratios are functions
2. -use trigonometric tables as an aid in drawing the graph of the sine and cosine functions for angle values which are acute.

LESSON OUTLINE AND CLASSROOM PROCEDURES

1. REVIEW

a. Basic Ideas:

-given a particular acute angle in a right triangle, the ratio opp:hyp is constant; adj:hyp is constant and opp:adj is constant.

-the values of these trigonometric ratios depend only on the measure of the acute angles.

b. Consequences:

1. -Given one side and one angle of any right triangle, the triangle can be solved.

   -illustrate with an example but do not complete the solution.

2. -Given two sides of any right triangle, the triangle can be solved.

   -illustrate with an example but do not complete the solution.

2. TRIGONOMETRIC RATIOS AND SETS OF ORDERED FACTS

-recall and illustrate how an equation such as $y = 2x + 4$ yields a set of ordered pairs which can be graphed.

-convince the students that in a similar way the trig ratios can yield a set of ordered pairs which satisfy an equation of the form y = sin x; y = cos x or y = tan x. These ordered pairs can be graphed on a rectangular grid.

3. GRAPH OF SINE FUNCTION

-distribute the rectangular grid labelled "Graphs of the Trig Functions" to the students.

-use the transparency of this grid (T18) to demonstrate how one or two points on the graph of the sine curve may be plotted.

4. ASSIGNMENT

a. Have the students complete the graph of y = sin x by plotting points using angles between 0° and 90° which are multiples of five.

b. Using the same set of axes have the students graph the cosine function for the same angle values.

LESSON 9 (C)

PURPOSE:

To review the trigonometry studied to date and to draw the graphs of the sine and cosine functions.

OBJECTIVES:

At the conclusion of this lesson the students should be able to:

1. -explain why trigonometric ratios are functions

2. -use trigonometric tables as an aid in drawing the graph of the sine and cosine functions for angle values which are acute.

3. -state the significance of the discovery of the rectangular coordinate system by Descartes to the study of trigonometry.

4. -appreciate that trigonometry is an evolving creation of the human intellect.

LESSON OUTLINE AND CLASSROOM PROCEDURES

1. REVIEW

Briefly review the trigonometry studied to date and include the contributions of Hipparchus and Ptolemy. Use the overhead transparencies previously provided for the review as outlined below.

a. Origins of Trigonometry

-at Alexandria - display transparency T1
-mention the major contributions of Hipparchus and Ptolemy - use transparencies T6 and T7.

b. Basic Ideas

-given a particular acute angle in a right triangle, the ratio opp:hyp is constant; adj:hyp is constant and opp:adj is constant.

-the values of these trigonometric ratios depend only on the measure of the acute angles.

c. Consequences

As a result of these basic ideas the following problem types can be solved:

1. -Given one side and one angle of any right triangle, the triangle can be solved.

   -briefly recall how Hipparchus used this fact to find the distance from the earth to the moon (display T13).

2. -Given two sides of any right triangle, the triangle can be solved.

   -briefly recall how Heron used this fact to show how a tunnel could be cut through a mountain by beginning at opposite sides.

2. DEVELOPMENT OF TRIGONOMETRY

Use transparency T19 as an aid to indicate (briefly) some of the major developments of trigonometry over the centuries. Note particularly the close connection between astronomy and trigonometry. Two French mathematicians played important roles in the development of trigonometry in the 15 and 16 hundreds. They were Vieta and Déscartes.

Vieta recognized the functional relationship between angles and trigonometric ratios of those angles and expressed this relationship in the form of an equation ($y = \sin x$; $y = \cos x$).

Déscarte's contribution was to provide the rectangular coordinate system which provided a method whereby a set of ordered pairs could be graphically displayed. Hence trigonometric Functions were first graphed in 1635. Stress the contribution of <u>Déscartes</u>.

3. TRIGONOMETRIC RATIOS AND SETS OF ORDERED PAIRS

-recall and illustrate how an equation such as $y = 2x + 4$ yields a set of ordered pairs which can be graphed.

-convince the students that in a similar way the trig ratios yield a set of ordered pairs which satisfy an equation of the form $y = \sin x$, $y = \cos x$ or $y = \tan x$. These ordered pairs can be graphed on a rectangular grid.

4. GRAPH OF THE SINE FUNCTION

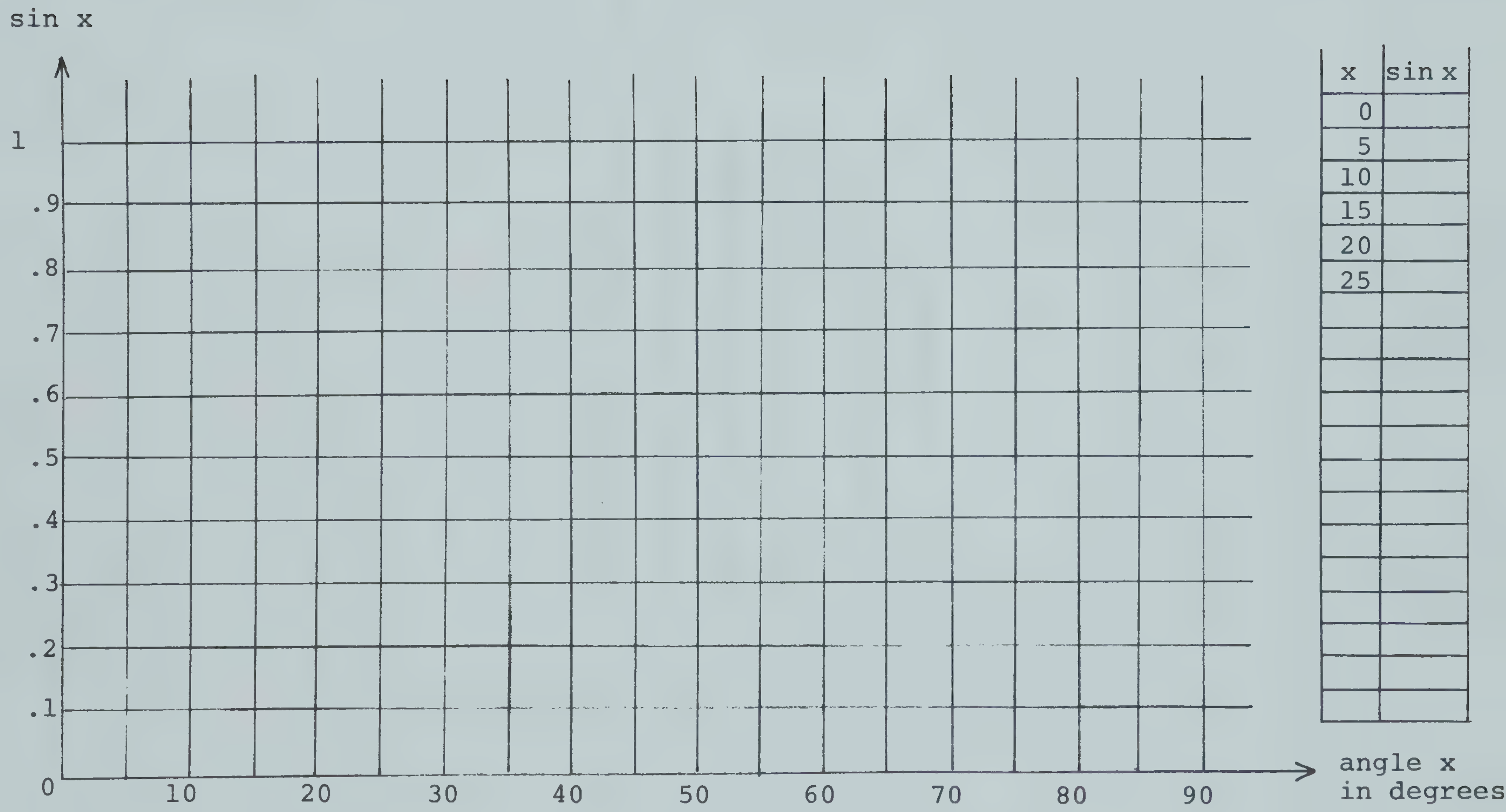
-distribute the rectangular grid labelled "Graphs of the Trig Functions" to the students.

-use the transparency of the grid (T18) to demonstrate how one or two points on the sine curve may be plotted.

5. ASSIGNMENT

a. Have the students complete the graph of $y = \sin x$ by plotting points using angles between 0° and 90° which are multiples of 5.

b. Using the same set of axes have the students graph the cosine function for the same angle values.



x	sin x
0	
5	
10	
15	
20	
25	

GRAPHS OF THE TRIG FUNCTIONS

y = sin x
y = cos x

APPENDIX 4

TESTING INSTRUMENTS

Student Achievement Subtests

Student Reaction Questionnaire

Teacher Reaction Questionnaire

Items of Student Reaction Questionnaire Categorized by Major Groupings

TRIGONOMETRY

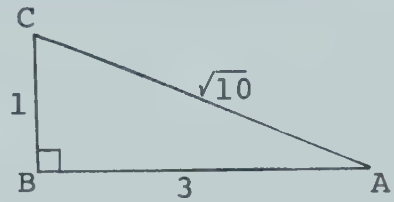
MATH 23 TEST 1

Do all work in the space provided on this question sheet. Draw diagrams where applicable.

1. a. In terms of the sides of a right triangle (opp, adj, hyp), define the sine, cosine and tangent of a given angle A.

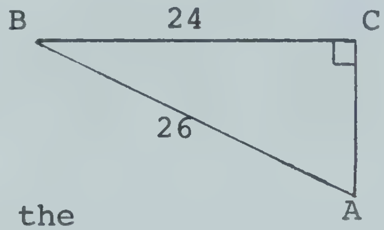
   sin A = cos A = tan A =

   b. Given right triangle ABC, find the value of $(\sin A)^2 + (\cos A)^2$



2. Using the diagram at the right, answer the following questions.

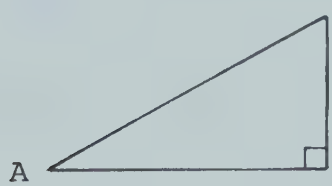
   a. Calculate the length of AC.



   b. Determine the value of each of the following (leave answers in fractional form):

   sin A = sin B = tan B =

3. If sin A = 3/5, calculate sin A + cos A.



4. Find the value of

   a. $\tan 89° + \cos 36°$

   $= \_\_\_\_\_ + \_\_\_\_\_$

   $= \_\_\_\_\_\_\_$

   b. $\dfrac{\cos 76°}{\sin 90°} = \_\_\_\_\_\_\_\_$

   $= \_\_\_\_\_\_\_\_$

5. From the top of a building 60 feet high, the angle of depression to a car on a road below is 24°. How far from the foot of the building is the car located (to the nearest foot)?

6. A ladder leans against a building and makes an angle of 62° with the ground. If the ladder is 20.6 feet long, at what height on the building does the ladder touch?

7. One end of a 150 foot guy wire is attached to the top of a radio tower and the other end is anchored in the ground. If the wire is to make an angle of 70° with the ground, how far from the base of the tower should the wire be anchored?

8. The ancient Greek city of Alexandria boasted a lighthouse which overlooked the Mediterranean Sea. From the top of the lighthouse the angle of depression to a ship at sea one mile from the base of the lighthouse was 4°. How high was the lighthouse? (1 mile = 5,280 feet).

TRIGONOMETRY

MATH 23 TEST 2

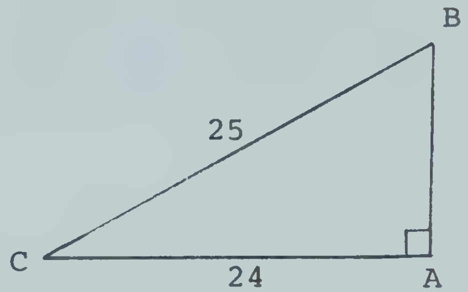
Note: Do all work in the space provided on this question sheet. Draw diagrams where applicable.

1. a. Find the value of sin 68° - cos 45°.

   b. Find A to the nearest degree in each of the following.

   cos A = .8090   tan A = .7526
   A = ______   A = ______

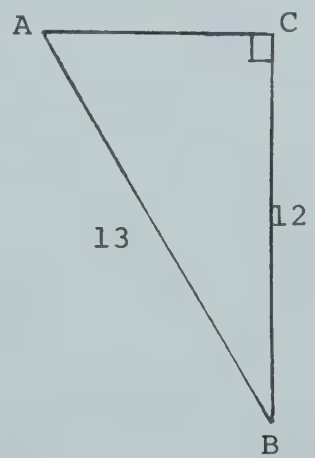
2. Using the triangle at the right, answer the following questions.



   a. Find the length of side AB.

   b. Find the value of cos C - sin C.

3. Solve the following right triangle; that is, determine the measure of all unknown sides and angles.

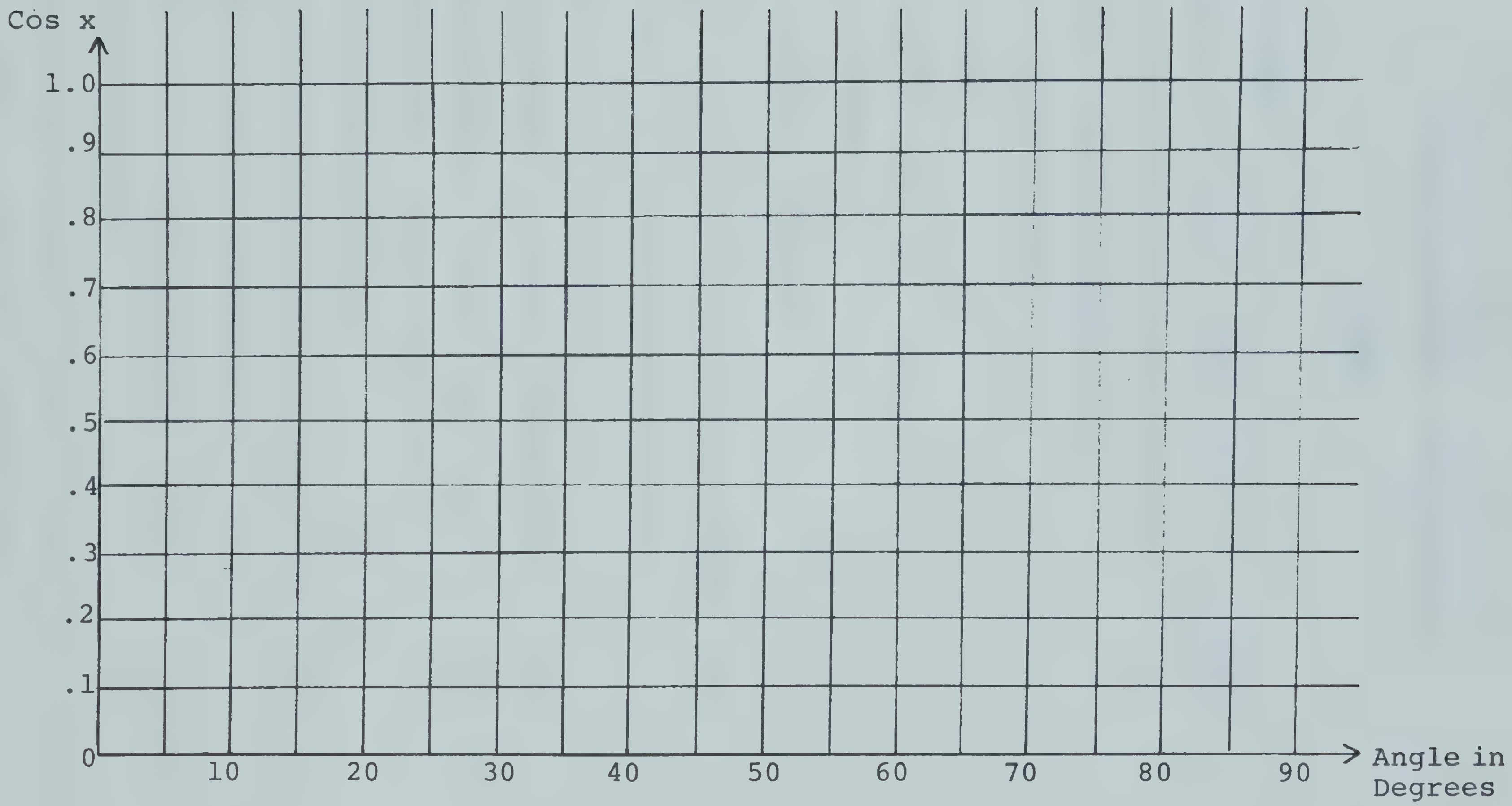


4. The C.N. Tower in Toronto is the tallest structure in the world. If the angle of depression to a stationary point 1 mile from the base of the tower is 19° calculate the height of the tower (to the nearest foot).

5. The Boeing 747 is the world's largest passenger aircraft. After takeoff this plane climbs to altitude at an angle of elevation of 2°. What will be the altitude of the aircraft 40 minutes after takeoff if the plane travels at 300 miles per hour?

6. A ladder 24 feet long is placed against a vertical wall. What angle does the top of the ladder make with the wall if the foot of the ladder is 5 feet from the base of the wall?

7. Draw the graph of y = cos x on the grid below.



x	5	10	15	20															
cos x																			

STUDENT REACTION QUESTIONNAIRE

PART I

TO THE STUDENT:

PLEASE CHECK EACH OF THE FOLLOWING STATEMENTS BY DRAWING A CIRCLE AROUND YOUR SELECTED RESPONSE.

THE KEY TO THE RESPONSES IS:

SA - STRONGLY AGREE

A - AGREE

U - UNDECIDED

D - DISAGREE

SD - STRONGLY DISAGREE

1. Only students who will use mathematics in later life should be required to study it in high school.	SA	A	U	D	SD
2. Trigonometry is useful mathematics	SA	A	U	D	SD
3. Mathematicians from the past and the problems they helped solve are of little interest to me.	SA	A	U	D	SD
4. I would like school better if I didn't take any mathematics.	SA	A	U	D	SD
5. Very little trigonometry is used in modern societies.	SA	A	U	D	SD
6. I believe mathematics would be more interesting to me if it related more to real people and real problems.	SA	A	U	D	SD

7. I found trigonometry more interesting than any other mathematics I have studied this year.	SA	A	U	D	SD
8. It is important that many people in our society know mathematics.	SA	A	U	D	SD
9. Every bit of mathematics that exists today was invented by somebody.	SA	A	U	D	SD
10. If I study mathematics again I hope it's not trigonometry.	SA	A	U	D	SD
11. The problem with learning trigonometry is that it is not useful.	SA	A	U	D	SD
12. Mathematics is not as important to people as art or literature.	SA	A	U	D	SD
13. I found trigonometry difficult to understand.	SA	A	U	D	SD
14. Trigonometry is a very worthwhile and necessary mathematics.	SA	A	U	D	SD
15. Trigonometry is not important for the advance of civilization and society.	SA	A	U	D	SD
16. I found trigonometry easier than most mathematics I have studied.	SA	A	U	D	SD

PART II

1. What did you like best about this unit of work on trigonometry?

2. What did you especially dislike about this unit on trigonometry?

3. What did you learn while studying this unit on trigonometry that surprised you most?

4. Comments:

TEACHER REACTION QUESTIONNAIRE

You recently participated (in the capacity of instructor) in an experiment involving two classes of Math 23 students taught under differing treatments: R (regular) and C (cultural). Relative to this experiment please answer the following questions.

1. What are your perceptions of the intent of treatment C?

2. How would treatment C have been improved in order to better achieve the intended and written aims of the treatment? Suggest possible additions and (or) deletions.

3. Do you feel treatment C was appropriate for Math 23 students of trigonometry?

4. What were your impressions of student reaction to treatment C?

5. Generally speaking, was treatment R a fair representation of what might ordinarily take place in Math 23 classes in trigonometry in your school?

6. What were your impressions of student reaction to treatment R?

7. Will you use any of the ideas developed in treatment C in future lessons in trigonometry?

8. Other comments.

ITEMS OF STUDENT REACTION QUESTIONNAIRE (PART I)
GROUPED INTO THE FOUR MAJOR CATEGORIES

DIFFICULTY ITEMS	13.	I found trigonometry difficult to understand.
	16.	I found trigonometry easier than most mathematics I have studied.
USEFULNESS ITEMS	2.	Trigonometry is useful mathematics.
	5.	Very little trigonometry is used in modern societies.
	8.	It is important that many people in our society know mathematics.
	11.	The problem with learning trigonometry is that it is not useful.
	14.	Trigonometry is a very worthwhile and necessary mathematics.
INTEREST ITEMS	1.	Only students who will use mathematics in later life should be required to study it in high school.
	4.	I would like school better if I didn't take any mathematics.
	7.	I found trigonometry more interesting than any other mathematics I have studied this year.
	10.	If I study mathematics again I hope it's not trigonometry.
CULTURAL ITEMS	3.	Mathematicians from the past and the problems they helped solve are of little interest to me.
	6.	I believe mathematics would be more interesting to me if it related more to real people and real problems.
	9.	Every bit of mathematics that exists today was invented by somebody.
	12.	Mathematics is not as important to people as art or literature.
	15.	Trigonometry is not important for the advance of civilization and society.